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On a parameter adaptive self-organizing system in the presence of large outliers in observations

Rimantas PUPEIKIS
Institute of Mathematics and Informatics
Akademijos 4, LT-08663 Vilnius, Lithuania
Vilnius Gedimino Technical University
Saulėtekio 11, LT-10223 Vilnius, Lithuania
e-mail: pupeikis@ktl.mii.lt

Abstract. The aim of the given paper is development of a minimum variance control (MVC) approach for a closed-loop discrete-time linear time-invariant (LTI) system when the parameters of a dynamic system as well as that of a controller are not known and ought to be estimated by processing observations in the case of additive Gaussian noise on the output with contaminating outliers uniformly spread in it. Afterwards, the current value of the control signal is found in each operation, and it is used to generate the output of the system. The results of numerical simulation by computer are presented and discussed here, too.

Keywords: self-tuning controller, the minimum variance control law, parametric identification, outliers.

1. Introduction

To provide self-tuning control of a real plant several ordinary control approaches, such as, MVC, generalized MVC (GMVC), incremental GMVC, and so on, are frequently used that take into consideration random disturbances affecting the process. The MVC and GMVC algorithms, as noted in [5], were the first that were designed specially for self-tuning applications and are now considered 'classical' formulations. The algorithms described there can be implemented as self-tuning controllers that underpin the design and development of a modern model based predictive control approach. On the other hand, it has been emphasized in [3] that in designing a robust control system, one ought to determine the type of uncertainties appearing in the system to be controlled. One of the main ones of them is the uncertainty arising in the output disturbance description of a plant model to be used. It is frequently assumed that system's output is affected by Gaussian disturbance. However, nonnormal noise, and particularly the presence of outliers, degrades the performance of a system acting in a closed-loop as well as the parametric identification of the same system. Besides, before calculating the value of the control signal it is important to find the values of the output that have not been harmed by outliers. To this end, we propose here ways and means how to solve these problems.

2. Statement of the problem

Assume that a system to be observed is a causal and LTI system with one output $\{y(k)\}$ and one input $\{u(k)\}$, expressed by the equation

$$y(k) = q^{-\tau} G_0\big(q^{-1};\ \theta\big)u(k) + \underbrace{H_0(q^{-1};\varphi)\xi(k)}_{v(k)}, \tag{1}$$

that consists of two parts: a system model $G_0(q^{-1};\theta) = B(q^{-1};\mathbf{b})A^{-1}(q^{-1};\mathbf{a})$ and a noise model $H_0(q^{-1};\varphi) = A^{-1}(q^{-1};\mathbf{a})$. Here $k = 1, 2, \ldots$ is the current number of observations of a respective signal, τ is a known time delay, $\theta^T = (\mathbf{b}^T, \mathbf{a}^T)$ are unknown parameter vectors to be estimated, q^{-1} is the backward time-shift operator such that $q^{-1}u(k) = u(k-1)$, and

$$\mathbf{b}^T = (b_0, b_1, \ldots, b_{n_b}), \quad \mathbf{a}^T = (a_1, a_2, \ldots, a_{n_a}) \tag{2}$$

are unknown parameters of respective polynomials.

Given the model (1) and measured data $\{u(1), \ldots, u(N), y(1), \ldots, y(N)\}$ and assuming that the white noise $\{\xi(k)\}$, $k = 1, 2, \ldots$ is really a sequence of independent identically distributed variables with an ϵ-contaminated distribution of the form $p(\xi(k)) = (1-\epsilon)N(0,\sigma_\mu^2) + \epsilon N(0,\sigma_\xi^2)$, and the variance $\sigma_\xi^2 = (1-\epsilon)\sigma_\mu^2 + \epsilon\sigma_\varsigma^2$, let us suppose that $\{\xi(k)\}$ is used to generate unmeasurable noise $\{v(k)\}$. Here $p\{\xi(k)\}$ is the probability density distribution of the sequence $\{\xi(k)\}$, $k = 1, 2, \ldots$; $\xi(k) = (1-\gamma(k))\mu(k) + \gamma(k)\varsigma(k)$ is the value of the sequence $\{\xi(k)\}$, $k = 1, 2, \ldots$ at a time moment k; γ is a random variable, taking values 0 or 1 with probabilities $p(\gamma(k) = 0) = 1-\epsilon$, $p(\gamma(k) = 1) = \epsilon$; $\mu(k), \varsigma(k)$ are sequences of independent Gaussian variables with zero means and variances $\sigma_\mu^2, \sigma_\varsigma^2$, respectively; besides, $\sigma_\mu < \sigma_\varsigma$; $0 \leqslant \epsilon \leqslant 1$ is the unknown fraction of contamination.

The aim of the given paper is to design a parameter adaptive self-organizing robust system with the MVC law in the case of additive noise $\{v(k)\}$, that contains large outliers and corrupts the output $\{y(k)\}$ of the LTI system.

3. Design of a self-organizing system

The MVC controller seeks to design the required control signal

$$u(k) = \frac{1}{b_0}\left\{\sum_{l=1}^{n_a} a_l y(k+\tau-l) - \sum_{i=1}^{n_b} b_i u(k-i) + r(k)\right\} \tag{3}$$

by minimizing with respect to $\{u(k)\}$ the quadratic performance function

$$J_{MV} = \lim_{N\to\infty} E\left\{\frac{1}{N}\sum_{k=0}^{N-1}[r(k) - y(k+\tau)]^2\right\}, \tag{4}$$

that refers to the variance of the error between set-point $r(k)$ and the controlled output τ-time steps in the future, $y(k+\tau)$ [5].

To implement the self-tuning MVC controller, it is necessary, firstly, to estimate LTI system's model unknown parameters (2) in such a noisy environment using robust M-techniques [1, 2, 4] and, secondly, to determine the value of control signal $u(k)$ in each current operation by substituting in (3) the values of abovementioned estimates $\hat{\mathbf{b}}^T = (\hat{b}_0, \hat{b}_1, \ldots, \hat{b}_{n_b})$, $\hat{\mathbf{a}}^T = (\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_{n_a})$. However, in such a case, the transfer of meanings of large outliers proceeds in random noise appearing in output observations. Therefore, in each current operation before calculating the value of the control signal $\{u(k)\}$ it is important to find the values of the output that have not been harmed by outliers. To this end, we propose here to generate an auxiliary output signal $\hat{y}(k+\tau)$ that will be without outliers. A self-organizing MVC strategy is achieved when estimation and control are carried out every current instant k simultaneously.

4. Simulation example

A closed-loop system to be analysed is described by a linear difference equation of the form

$$\begin{aligned}\left(1 + a_1 q^{-1} + a_2 q^{-2}\right) y(k) &= q^{-1}\left(b_0 + b_1 q^{-1}\right) u(k) \\ &\quad + \left(1 + a_1 q^{-1} + a_2 q^{-2}\right)^{-1} \xi(k), \end{aligned} \tag{5}$$

while the MV controller design equation is

$$u(k) = \left(a_1 y(k) + a_2 y(k-1) - b_1 u(k-1) + r(k)\right)/b_0. \tag{6}$$

Here $a_1 = -1.5, a_2 = 0.7, b_0 = 1$ and the value of coefficient b_1 varries from 0.5 to 0.6 over 400 observations. The output $\{y(k)\}$, $k = 1, 2, \ldots, 400$ of the closed-loop system will be observed under the additive noise $\{v(k)\}$, $k = 1, 2, \ldots, 400$ in the presence of large outliers.

Firstly, the initial values of estimates $\hat{a}_1$, $\hat{a}_2$, $\hat{b}_0$, $\hat{b}_1$ of the true parameters a_1, a_2, b_0, b_1 of Eq. (5) were calculated by the ordinary LS with Mallow's estimator using 23 pairs of observations of $u(k), y(k)$. Secondly, we recursively calculate the estimates $\hat{a}_1, \hat{a}_2, \hat{b}_0, \hat{b}_1$ of the same parameters a_1, a_2, b_0, b_1 by processing $k = 24, 25, \ldots, 400$ observations of the control signal $\{u(k)\}$ and the output $\{y(k)\}$ in each current iteration, using two S-algorithms with a version of Shweppe's GM-estimator [1]. Thus, the output signals $\{y(k)\}$ of the same system (5) to be processed by both algorithms were different and generated in two ways (see Figs. 1a, b, e, f):

$$\begin{aligned} y(k) &= y_*(k) + \left(1 + a_1 q^{-1} + a_2 q^{-2}\right)^{-1} \xi(k), \\ y_*(k) &= q^{-1}\left(b_0 + b_1 q^{-1}\right) u(k) - \left(a_1 q^{-1} + a_2 q^{-2}\right) y_*(k), \end{aligned} \tag{7}$$

with

$$u(k) = \left[\hat{a}_1 y(k) + \hat{a}_2 y(k-1) - \hat{b}_1 u(k-1) + r(k)\right]/\hat{b}_0, \tag{8}$$

and with

$$u(k) = \left[\hat{a}_1 \hat{y}(k) + \hat{a}_2 \hat{y}(k-1) - \hat{b}_1 u(k-1) + r(k)\right]/\hat{b}_0, \tag{9}$$

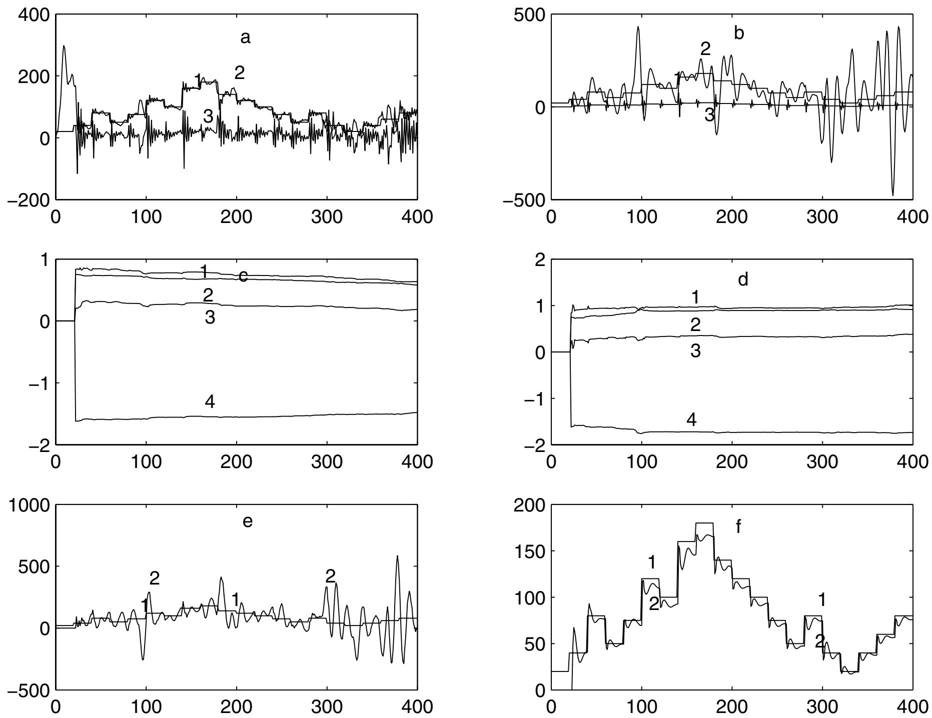

Figure 1. The signals and parametric identification results in the presence of additive noise with outliers in it dependent on the number of recursive iterations: x-axis – numbers of iterations, y-axis – meanings of the signals (a, b, e, f) and estimates (c, d); a, b, e, f are signals: the reference signal $\{r(k)\}$ – 1, output signals: $\{y(k)\}$ – 2 (a, b), $\{y_*(k)\}$ – 2 (e, f), the control signal $\{u(k)\}$ – 3, respectively, the current values of which have been determined as follows: by substituting the observed noisy values of $\{y(k)\}$ in formula (8) (a, c, e), and by substituting noiseless values of the auxiliary signal $\{\hat{y}(k)\}$ in formula (9) (b, d, f); in c, d: $\hat{b}_0, \hat{b}_1, \hat{a}_1, \hat{a}_2$ – 1, 3, 4, 2, respectively. The fraction of contamination $\epsilon = 0.1$.

where

$$\hat{y}(k) = q^{-1}\big(\hat{b}_0 + \hat{b}_1 q^{-1}\big)u(k) - \big(\hat{a}_1 q^{-1} + \hat{a}_2 q^{-2}\big)\hat{y}(k), \tag{10}$$

correspondingly, because in each recursive iteration $k = 24, 25, \dots, 400$ the current value of the control signal $\{u(k)\}$ is generated according to (8) (here the observed noisy values of $\{y(k)\}$ are substituted), and according to (9) (here the values of the noiseless auxiliary signal $\{\hat{y}(k)\}$ are applied). In both cases the current estimates $\hat{a}_1, \hat{a}_2, \hat{b}_0, \hat{b}_1$ are used. Afterwards, two different current values of the output signal $\{y(k)\}$ are calculated by formulas (7), where different current values of $\{u(k)\}$ are used. Then, different values of $\{u(k), y(k)\}$ are processed separately, in calculating the estimates $\hat{a}_1, \hat{a}_2, \hat{b}_0, \hat{b}_1$ of true values of the parameters a_1, a_2, b_0, b_1.

It follows that the accuracy of estimates $\hat{a}_1, \hat{a}_2, \hat{b}_0, \hat{b}_1$ of the parameters a_1, a_2, b_0, b_1, obtained by two separate acting recursive procedures with the version of Shweppe's GM-estimator, decreases when the amplitudes of values of the additive noise $\{v(k)\}$

with outliers in it are increasing (Figs. 1c, d). In such a case, the true output signal $\{y_*(k)\}$ (7) does not track the reference one, if the control signal $\{u(k)\}$ is calculated according to (8) (see Fig. 1e). Therefore it is important for calculating current values of the control signal $\{u(k)\}$ to use formulas (9)–(10) because, in such a case, the output signal $\{y_*(k)\}$ of form (7) tracks the reference one (Fig. 1f).

5. Conclusions

Despite that the MVC approach has been worked out for a random disturbance generated from the statistically independent and stationary sequence, it appears to be also applicable in the presence of large, but rare outliers in output observations in case the robust recursive parametric identification algorithms are used. One can state that the use of auxiliary signal $\{\hat{y}(k)\}$ allowed us to increase the efficiency of an adaptive LTI system with a self-tuning MVC controller significantly.

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REZIUMĖ

R. Pupeikis. Apie parametrų atžvilgiu adaptyviąją savitvarkę sistemą, esant didelės amplitudės triukšmų impulsams stebėjimuose

Straipsnyje nagrinėjama grįžtamojo ryšio diskrečiojo laiko tiesinė dinaminė sistema, veikianti pagal mažiausiosios dispersijos valdymo (MDV) dėsnį. Tariama, kad dinaminės sistemos bei reguliatoriaus koeficientai nėra žinomi ir turi būti įvertinami pagal įėjimo bei užtriukšminto išėjimo stebėjimus. Triukšmuose atsitiktiniais laiko momentais pasirodo reti didelės amplitudės impulsai. Siūloma sistemos parametrams įvertinti taikyti rekurentinius patvariuosius algoritmus, o jos valdymo signalo einamajai reikšmei gauti – papildomo modelio išėjimą, laisvą nuo triukšmų. Pateikti II eilės MDV sistemos modeliavimo rezultatai.

Raktiniai žodžiai: grįžtamojo ryšio sistema, mažiausiosios dispersijos valdymo dėsnis, patvarieji algoritmai, stebėjimai, triukšmai.