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Light beam phase retrieval in nonlinear media: a computer simulation

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1. Introduction

Phase retrieval problem have attracted increasing attention in various fields of astronomy and physics such as crystallography, electron microscopy or wave front sensing [1–3].

In a simplified way, the light wave transport in two-dimensional nonlinear space could be described with the following nonlinear Schrödinger equation:

$$\frac{\partial u}{\partial z} = ia\frac{\partial^2 u}{\partial x^2} + ib|u|^2 u. \tag{1}$$

The solution of this equation is a function $u(t,x)$ and its absolute value could be directly measured but its phase could not be obtained in a physical experiment. Usually, the phase of the light wave is retrieved by using the Gerchberg–Saxton algorithm and measuring the light intensity in two parallel planes – $|u1(x)|$ and $|u2(x)|$. Fig. 1 shows a typical experimental scheme [4].

The most commonly used experimental setup has lens between the planes P1 and P2 and the light transport transformation is approximated with a Fourier transform which in turn is replaced by a discrete Fourier transform. Note that its calculation does not require big computing power, therefore, many of the Gerchberg–Saxton algorithm iterations could be calculated in a short period of time. In the nonlinear media, light wave transport is described by Eq. 1, thus, cannot be approximated with the Fourier transform.

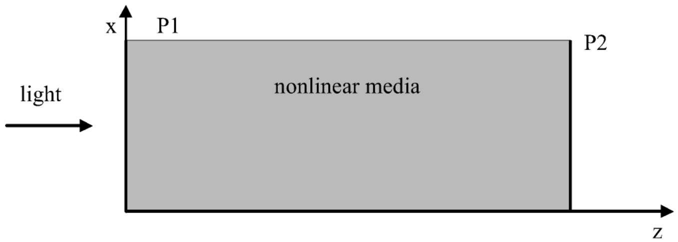

Fig. 1. Typical layout of the experiment (here P1 and P2 are the intensity measuring planes).

2. Computer simulation setup and results

We performed a computer simulation series to investigate the application of the Gerchberg–Saxton algorithm for the light phase retrieval in nonlinear media. Our main tasks were to examine how the algorithm convergence depends on iteration count, how the algorithm error (efficiency) depends on light transport medium properties (values of the coefficients a and b), and the algorithm resistance to noise in measurement data when light transport is calculated by numerically solving Eq. 1.

Our computer simulation plan is as follows.

1. A function was selected as a light intensity measurement data on plane P1.
2. Eq. 1 was solved with finite differences method by using this complex function as initial conditions and natural boundary conditions. The solution was interpreted as light intensity measurements on plane P2.
3. Fixed number of the Gerchberg–Saxton algorithm iterations was executed on these data.
4. Phase retrieval error was evaluated by comparing retrieved phase values to the original phase values.
5. A series of such simulations were performed by modifying the Schrödinger equation coefficients and the number of algorithm iterations to investigate the phase retrieval error dependency on these properties.

Computer simulation parameters: the nonlinear Schrödinger equation was solved numerically by the Krank-Nicholson finite differences scheme [5] (for linear part of equation) and iteration process for a nonlinear part of the equation, nonlinear equation solution accuracy – $\varepsilon_{non-linear} = 10^{-9}$, solution area – $z = [0..0.5]$, $x = [0..20]$, grid step size – $hz = 0.01$, $hx = 0.01$. Phase retrieval error (efficiency) was measured as phase differences norm $\|\cdot\|_2$ for the points where the light intensity absolute value was greater than u_{min}:

$$\delta_{err} = \min_{\alpha} \delta(\alpha) = \begin{cases} \left\| \varphi_1(x) - \varphi_{1orig.}(x) - \alpha \right\|_2, & |u_1(x)| \geqslant u_{min}, \\ 0, & |u_1(x)| < u_{min}, \end{cases} \tag{2}$$

where $\|\cdot\|_2$ is a vector norm, defined as $\|\vec{x}\|_2 = \sqrt{\frac{1}{n}\sum_{i=1}^{n}|x_i|^2}$; here $\varphi_{1orig.}(x)$ is the original phase of the function $|u_1(x)|$. The addition of a constant value to the complex function phase does not affect the absolute value of this function, therefore (since only the absolute value measurements are used for phase retrieval) the Gerchberg–Saxton algorithm retrieves phase with some constant delta if compared with the original phase. Constant phase delta is eliminated in phase retrieval error (efficiency) evaluation. For this simulation we choose $u_{min} = 0.01$, $u_1(x) = \mathrm{e}^{-(x-10)^2 + i\cdot\sin(x-10)\cdot \mathrm{e}^{-(x-10)^2}}$.

In the first computational experiment, we investigated how the algorithms error (efficiency) depends on the number of iterations. The following coefficient values were used in this simulation: $a = -1$, $b = 0; -2; -10$. The results are shown in Fig. 2.

The first simulation revealed that the phase retrieval error rapidly decreases during the first 100 iterations and after 1000 it achieves almost optimal value. In practical applications, it is enough to run 100–1000 iteration. Simulation results also revealed that

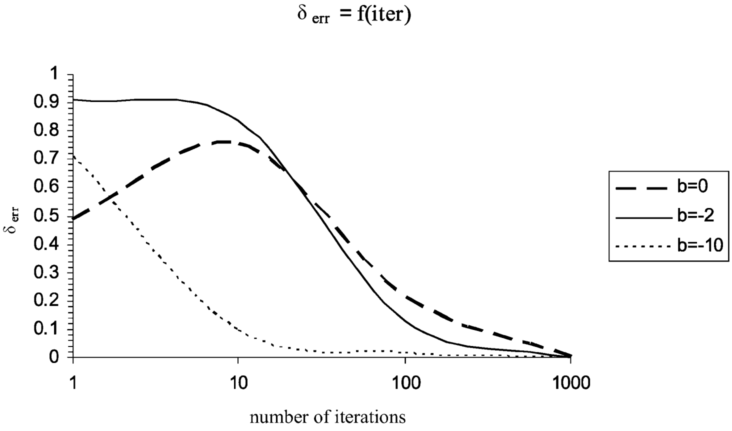

Fig. 2. Phase retrieval error (efficiency) versus iteration count.

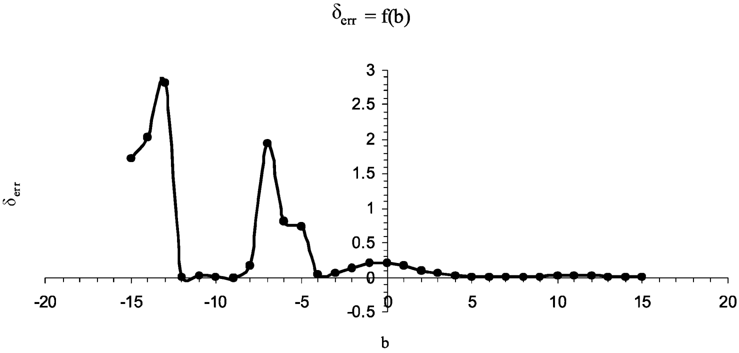

Fig. 3. Phase retrieval error (efficiency) versus media nonlinearity (b).

phase retrieval error (efficiency) depends on the nonlinearity of the light distribution media. Phase retrieval performance is an important factor in practical applications, our simulation performance results were as follows: 11000 iterations of the algorithm took 2291 sec., 100 iterations 234 sec., 10 iterations 27 sec., and 1 iteration 5 sec. The simulation was performed on a 2GHz Athlon64 machine. Thus, the Gerchberg–Saxton algorithm could be applied in practice even when the wave transport equation is solved with finite differences method.

The second computational experiment was designed to analyze the dependency of the phase retrieval error (efficiency) on media nonlinearity. During this simulation, the coefficient a value was fixed at -1, the b value varied from -20 to 20, and the phase retrieval iteration count was 100. The results are shown in Fig. 3.

It is clearly visible that phase retrieval error (efficiency) is different for positive and negative b values and the positive b values are more favorable than negative values. It could be explained by the light transport differences for different b values, since for

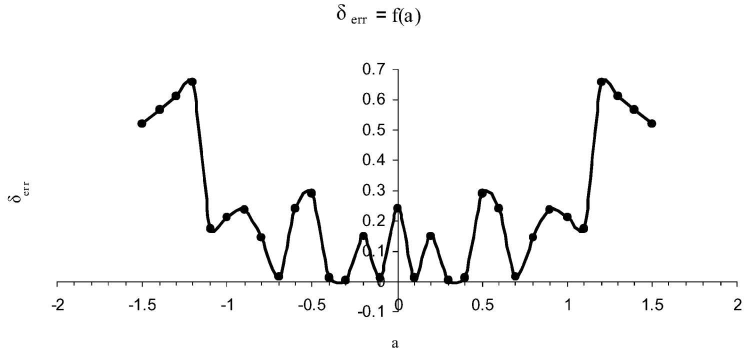

Fig. 4. Phase retrieval error (efficiency) versus coefficient a.

positive b values the light beam defocuses and spreads over the area where $|u_2(x)| > 0$. For negative b values the light beam defocuses and shrinks the area where $|u_2(x)| > 0$. The inefficiency peaks for negative b values cannot be easily explained and, perhaps, requires a separate investigation.

An additional computational experiment was performed to determine the phase retrieval error (efficiency) dependency on coefficient a. During this simulation, b was fixed to 0 and the a value varied from -1.5 to 1.5 (greater values expanded equation solution beyond solving area). For the phase retrieval iteration count equal to 100, the experimental results are shown in Fig. 4. They reveal that a values have a great influence for the phase retrieval error.

An additional simulation was designed and performed to analyze the dependency of the phase retrieval error (efficiency) on the noise in measured data. Some amount of uniformly distributed noise was added to the original light intensity values for planes P1 and P2. The amplitude of the noise varied during the simulation and imitated noise in physical measurements. The simulation results with $a = -1$ and $b = 0; -2$, are shown in Fig. 5.

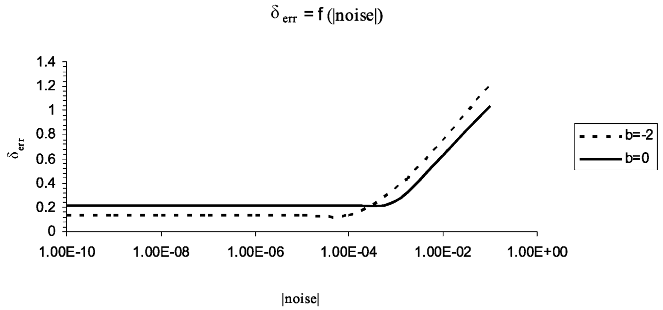

Fig. 5. Phase retrieval error versus noise amplitude.

As a conclusion, we can state that the noise with amplitude lower than 10^{-4} does not affect phase retrieval error (efficiency). For higher noise levels, phase retrieval error is linearly proportional to the logarithm of noise amplitude.

3. Conclusions

A detailed analysis of our computational experiment allows us to make the following conclusions:

1. The Gerchberg–Saxton algorithm converges well in nonlinear media and good results can be achieved already after 100 iterations; after 1000 iterations the phase retrieval error is minimal.
2. Noise in initial intensity measurements lower than 0.01% (from the maximum intensity value) does not affect the phase retrieval error (efficiency); acceptable results can be achieved with noise levels up to 1%.
3. The phase retrieval error (efficiency) is affected by light transport medium properties (the values of a and b).
4. The performance of modern personal computers makes possible practical phase retrieval in nonlinear media when the light transport is calculated by solving the nonlinear Schrödinger equation by numerical methods.

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REZIUMĖ

M. Puida, F. Ivanauskas. Šviesos pluošto fazės atstatymo netiesinėje aplinkoje kompiuterinis modeliavimas

Šiame straipsnyje pristatomas šviesos spindulio fazės atstatymo kompiuterinis modeliavimas, šviesai sklindant netiesinėje aplinkoje. Fazės atstatymui pritaikant Gerchberg–Saxton algoritmą bei baigtinių skirtumų metodą. Modeliavimo rezultatai parodė, kad fazės atstatymas netiesinėje aplinkoje gali būti taikomas praktikoje, taip pat nustatyta fazes atstatymo kokybės priklausomybė nuo algoritmo iteracijų skaičiaus, netiesinės aplinkos savybių bei triukšmo lygio matavimuose.