Liet. matem. rink, **46***, spec. nr.*, 2006, 321–327

Determination and simulation of stimulated dynamics

Robertas ALZBUTAS (LEI, KTU), Vytautas JANILIONIS (KTU)
e-mail: robertas@mail.lei.lt, vytautas.janilionis@ktu.lt

1. Introduction

Probabilistic dynamics and dynamic reliability

A number of different methodologies were proposed in order to deal with the time that elapses during the evolution of dynamics. Most known theoretical background of these methodologies to treat and analyse dynamic systems was based on Markovian framework. For instance, the Theory of Probabilistic Dynamics (TPD) [1] was extensively investigated in order to perform analytical modelling related to the analysis of systems reliability and safety. Dynamic reliability techniques [2] have been developed in order to study the reliability parameters of complex dynamic systems having continuous processes and discrete failure events. In dynamic reliability theory, the concept of reliability includes the interaction existing between the sequence of dynamics and events, such as the crossing of the border of a safety domain in the space of the physical variables, as well as the transitions between dynamics.

Recently the Theory of Stimulated Dynamics (TSD) is developed for analytical modelling of hybrid (continuous-discrete) systems. The theory at first deals with instantaneous and random variations of process variables; then, it introduces the concept of stimulus and how it can be implemented. A non-Markovian treatment is provided in order to adapt TPD for practical applications, mostly in the context of Probabilistic Safety Assessment (PSA) [3]. The development of Stimulus Driven Theory of Probabilistic Dynamics (SDTPD) or Theory of Stimulated Dynamics (TSD) as well as related methods and simulation methodologies has been agreed as a basis for research continuation in the perspective of it applications for PSA and severe accident management.

Issues of simulation and aggregate approach

The analytical modelling and simulation methods are used separately as a rule in order to analyse the physical processes and random events. However, in the currently used classical PSA, the lack of treatment of dynamic interactions between the physical processes and random events causes the issues to model and latter on to simulate imprecise time delays in the actuation of control/protection signals, distributed parameters in the dynamics, uncertain limits of safety domain in the process variable space and etc. Thus, for more complex analysis there is obvious need to integrate different modelling approaches and apply the combined modelling (analytical modelling and simulation), which enables to join both methods advantages and avoids many of their disadvantages.

In general, there are a lot of mathematical modelling schemes, which can be used separately or integrated in order to extend possibilities of modelling and analysis. The aggregate approach and the method of control sequences have been investigated and widely used at Kaunas University of Technology (KTU). Using this approach the simulation of dynamic systems [4] and integration of modelling methods [5], [6] was also considered. According to this approach the investigated objects are presented as the set of interacting Piecewise Linear Aggregates (PLAs) [7]. The method of control sequences is used for the aggregate specification. Initially, PLA formalism was mainly used for discrete event system specification and analysis of distributed systems [8]. In our case, applying the advantages of PLA, the focus is set on simulation and analysis of hybrid systems considering the stimulated dynamics and interactions with various events.

2. Stimulated dynamics

At first, let us determine analytically the basic ingredients of stimulated dynamics and consider them in relation to each other. Events are associated to an instantaneous change of the dynamics due to stimulus. System dynamics is determined by the law of process variables evolution, which can be indexed by an integer $i \in N$. Process variables $\bar{x}$ can be governed by a set of deterministic equations, e.g., $d\bar{x}/dt = f_i(\bar{x}),\ \bar{x}(0) = \bar{x}_0,\ \bar{x} \in R^N$. Event e is defined as transition of dynamics, in particular case from dynamics i to dynamics j at time t, i.e., e: $C \to C$. Random event is the event whose occurrence is related to complex nature, which is modelled stochastically. An example of random event is a time distributed failure occurrence. Deterministic event is induced by the deterministic rules. An example of such an event can be related to time moment when a threshold pressure or temperature is reached and safety functions is activated. Paths of dynamics or evolution of process variables between transitions are associated to deterministic transients. Paths depend on initial and boundary conditions, which are associated to the initial states of process variables. Transitions are associated to significant changes in dynamic status, i.e., the end of one dynamics and the beginning of next one. Sequence of transitions is related to the sequence of events, part of which can be introduced artificially just for simulation purposes. To describe transition (in particular case due to event e) from dynamics i to dynamics j at time t, let us define the transition rate between dynamics $j \to i$ as $p(j \to i|\bar{x})$, which characterizes the part of the sequence of transitions (or the sequence of events if all transitions are related to events) corresponding to the transition between two dynamics $j \to i$. Then the total transition rate out of dynamics j is equal to the sum of all transition rates out of dynamics j to any $i \neq j$:

$$\lambda_j(\bar{x}) = \sum_{i \neq j} p(j \to i|\bar{x}).$$

All the information on the any dynamics i is accumulated into the probability density function $\pi(\bar{x}, i, t)$ of the dynamics i at time t and with process variables in $d\bar{x}$ about $\bar{x}$.

Before description of sequence of events, two transition densities related to any dynamics i can be also introduced:

1. Outgoing density out of dynamics i to any $k \neq i$ at $(\bar{x}, t)$

$$\psi(\bar{x}, i, t) \equiv \sum_{k \neq i} p(i \to k|\bar{x})\pi(\bar{x}, i, t) = \lambda_i(\bar{x})\pi(\bar{x}, i, t).$$

2. Ingoing density into dynamics i from any j $\neq$ i at $(\bar{x}, t)$

$$\varphi(\bar{x}, i, t) \equiv \sum_{j \neq i} p(j \to i|\bar{x})\pi(\bar{x}, j, t) = \sum_{j \neq i} \hat{p}(j \to i|\bar{x})\psi(\bar{x}, j, t).$$

To describe event in relation to stimulus, there is need to note that stimulus covers any situation that potentially causes, after a given time delay, an event to occur (see Fig. 1). In the usual formulations of the theory of probabilistic dynamics the change in the dynamic behaviour of the physical process variables occurs with no delay after the solicitation causing a branching in the continuous event tree. The main concept introduced in stimulus driven theory of probabilistic dynamics is that of stimulus, which must take place prior to the actual transition between two dynamics, i.e., system configurations corresponding to different dynamic evolutions.

Let Φ be the set of all stimuli to be accounted for in the process evolution following the occurrence of a given event related to the transitions between dynamics. Denote by $f_i^F(\tau_F; \bar{u})$ the probability density function of activating particular stimulus $F \in \Phi$ at states $\bar{g}_i(\tau_F, \bar{u})$ after time τ_F spent in dynamics i which was entered at state $\bar{u}$. Also define $h_{ik}^F(t_d; \bar{u})$ – probability per unit time of having a time delay $t_{dk} = t_F - \tau_F$ between stimulus F activation at time moment τ_F and occurrence of event, i.e., transition of dynamics $i \to k$, if stimulus F was activated at states $\bar{u}$. $h_i^F(t_d; \bar{u}) = \sum_{k \neq i} h_{ik}^F(t_{dk}; \bar{u})$ is probability density function of the delay t_d between stimulus F activation at time moment τ_F and in the same conditions leaving dynamics i at time moment t_F.

Sequence of events consists of events, with system dynamics in between. Every sequence of events should be unique, but can lead to the same consequence. Sequence of event is an instance of the system status evolution through time line, i.e., $E = e \times T$. Space of sequences of events is the set of all possible event sequences, i.e., $S_E = \{E_i\}$.

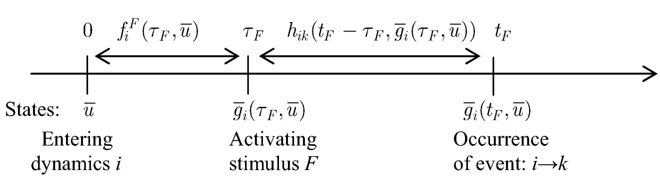

Fig. 1. Stimulus F activation and delay before event occurrence.

Event sequences in an event sequence space are considered to be mutually exclusive, even though they may partially overlap, since they are assumed to be related to dynamics, which originate from a single initiating state of the process variables and change at each branching point. Branching point represents a time moment when dynamics changes due to possible occurrence of event. At each branching time moment the dynamics may have one or more branches, which correspond to the following dynamics, depending on previous dynamics and possible occurrence of different events. Scenario s_i is a simplified representation of sequences with some common features. These features as example may concern the classification of events or/and separation according to timing of events. The sequences belonging to a same scenario are therefore considered to be similar, to the extent that they share the features implied by the scenario. Rare scenario from definition point of view is related to the rareness of sequences.

The practical simulation of sequences asks for the introduction of some kind of simulation approach. In the case of stimulus driven dynamics, the proposed approach considering discrete time moments restricts the amount of possible branching causes and branching points in time. After the occurrence of an initiating event, the related evolution laws of the process variables are considered. The corresponding deterministic process evolution defines a branch, from which all stimuli with related events causing the system to branch off are accounted for at those user-specified discrete time intervals. The same branching process is carried on until considered process variables reach final absorbing end states (consequence expressed as a damage state or as a steady safe situation). The frequency of all sequences of dynamics can be calculated as they develop in the simulation.

3. Aggregate based simulation

If the aggregate mathematical modelling scheme (A-scheme) is applied for system specification and simulation, then the system is presented as a set of interacting piece-linear aggregates. In general macro modelling stage aggregates purposes and relationships are described. Later on, in micro modelling stage the functioning of each aggregate is specified. Each PLA is taken as an object defined by a set of states Z, input variable $X_d \in X$ and output variable $Y_d \in Y$. These variables with external events e' and internal events e'' and control sequences ξ are considered to be time functions. Each aggregate is functioning in a set of time moments $t \in T$. Apart from these sets, transition H and output G operators must be defined as well. The state $z \in Z$ of piece-linear aggregate can be related to the state of a piece-linear Markov process, i.e.: $z(t) = (v, z_v(t))$, where v is a discrete state component taking values on a countable set of values; and $z_v(t)$ is a continues component can be related to discrete component v and comprising time dependant values.

If system functioning can be expressed algorithmically, then the usual A-scheme is suitable to be used for simulation. However, if functioning of even one part of system, or its control condition as well as characteristic is described with differential equations or derivatives of functions, then usual A-scheme should be extended and special dynamic aggregate could be used in order to get values of process variables $\bar{x}$ in every analyzed time moment. The proposed dynamic aggregate is related to the dynamic mathematical modelling scheme (D-scheme). The application of D-scheme is based on analytical specification using differential equations and their systems. For practical simulation using this scheme there is developed a tool [6], which could solve general nth order differential equations and general systems of first-order differential equations. The solution of equations is expresses as function using Taylor row.

The simulation of stimulated dynamics is proposed to be realized using the dynamic aggregate and introducing other special aggregates for integration of D-scheme in to A-scheme. The integration of different modelling schemes is related to specification of interaction between events related to stimuli and process considered in hybrid system. Specifically, the simulation of continuous process variables $\bar{x}$ can be realized using the dynamic aggregate named Process. The control of transitions and various scenarios related to random events simulation can be realized with special observing/informing and managing aggregates: Observer and Manager (see Fig. 2).

A conceptual model and description of each aggregate (e.g., Manager, Observer and Process) can be presented in terms of A-scheme according to this approach. The simulation of each part of model can be formally described according to the steps of formal aggregate specification. Below there is an example of aggregate specification, which presents the formal description of aggregate Process in accordance to the notation presented in above and can be used in TPD.

Aggregate: Process

1. Input variables set: $X = \{X_1, X_2\};\ X_1 = (\bar{x}_0, i),\ X_2 = t;$
2. Output variables set: $Y = \{Y_1\};\ Y_1 = \bar{x}$, where $\bar{x} = \bar{g}_i(t, \bar{x}_0);$
3. External events set: $E' = \{e'_1, e'_2\};\ X_d \to e'_n,\ d \in \{1, 2\}$ and $n = d;$
4. Internal events set: $E'' = \emptyset;$
5. Aggregate states set: $Z = \{z_i(t)\} = \{(v_i,\ z_v(t))\},$
   Discrete component: $v_i = (\bar{x}, i),$
   Continuous component: $z_v(t) = \bar{g}_i(t, \bar{x}_0);$
6. Control sequences: $\{\xi_i\} = \emptyset;$
7. Initial state: $z_0(t_0) = (v_0,\ z_v(t_0)),\ v_0 = (\bar{x}_0, 0),\ z_v(t_0) = \bar{g}_i(0, \bar{x}_0);$
8. Operators: transition $H(e_i)$& output $G(e_i)$
   $H(e'_1)$: $Z = \{(v_i, z_v(t))\},\ v_i = (\bar{x}_0, i),\ z_v(t) = \bar{x}_0;$ G(e$'_1$): $Y = \emptyset;$
   $H(e'_2)$: $Z = \{(v_i,\ z_v(t))\},\ v_i = (\bar{x}, i),\ z_v(t) = \bar{x};$ G(e$'_2$): $Y = \{Y_1\},\ Y_1 = \bar{x}.$

The internal events set E''can be used in the Process aggregate in order to introduce all stimuli without additional aggregates. However in this case the stimulus will be

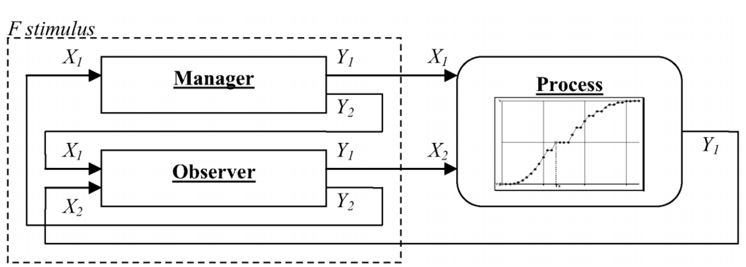

Fig. 2. Aggregates for events and process interaction.

only in a semi-Markovian framework. In order to employ the stimulus-driven Non-Markovian framework [3], i.e., to keep and evolve according information in history, two aggregates, namely Manager and Observer, can be introduced for each stimulus.

The abilities of adaptive managing of the process and changing of its functioning can be developed. This can be done using conditional control sequences ξ and interacting PLAs. For example, it is possible the modelling of process, which at first is based on one law, and in case, it reaches the defined state – on another law. Variables of continuous process with changeable laws can be defined using differential equation and their systems.

As for simple example let us consider the process with two parts, i.e., two different dynamics. In the both parts, where respectively $t \in (T_0, T_1)$ and $t \in (T_1, T_2)$ process is described by two differential equations, which depends on the time t and changeable parameter K. The parameter K represents the intensity of process and itself can be dependable on process initial parameter L (e.g., $K = 3 \cdot \sqrt{L}$). The first part of process is based on the equation $dy/dt = K \cdot t$, which common solution is $y = K/2 \cdot t^2 + C$. Let us assume $C = 0$, i.e., it is valid condition $y = 0$, when start time moment $t = 0$. The stimulus F_X and transition between dynamics is related to the change of process at the time moment $T_X = T_1$, when some limit state (for instance $y = L/2$) has been reached. As example the law of the second part of process could be equation $dy/dt = K \cdot (T_x - t)$, which common solution is $y = K(T_x \cdot t - t^2/2) + C$. If again $C = 0$ and following dynamics start time moment $t = 0$, then process would continue starting at the same state $y(T_X)$. Finally, we can assume that this dynamics will finish at time moment T_2, when the process state is close to the initial parameter, for instance L. The similar process will be restarted, when the initial stimulus F_0 will appear again. In general, each time moment $T \in \{T_0, T_1, T_2\}$ and associated stimuli $F \in \Phi$ can be related to the random delays. An example of computer simulation is presented in the Fig. 3.

According to the each stimulus F aggregate Manager initiates an event to manage the evolution of dynamics, i.e., to start, change or finish the process. The Observer checks the dynamics and limits in order to get the time moment needed for stimulus F generation. In this case, the time delay after stimulus F generation can be simulated

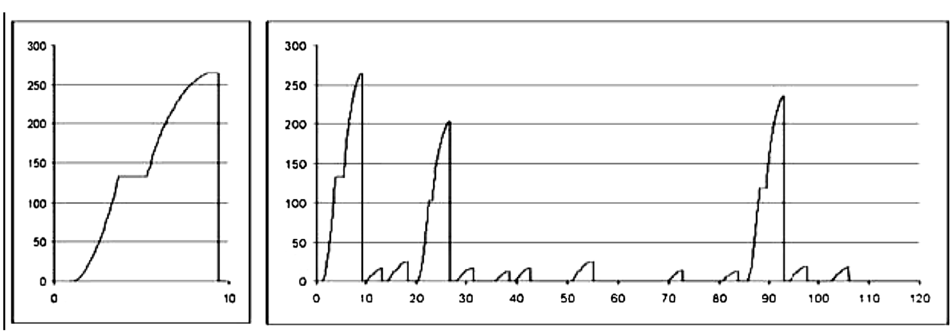

Fig. 3. An example of process simulation considering different time intervals.

in aggregate Manager adding internal event with related control sequence. The control sequence is used in order to define the actual time for the dynamics initiation.

Conclusions

The elements related to the stimulated dynamics are determined and aggregate based simulation approach is applied in order to consider the probabilistic dynamics and dynamic reliability. Approach based on integration of D-scheme in to A-scheme permits to use the same formal specification for simulation of continuous part of hybrid systems. The decomposition of simulation tasks enables to simulate the dynamic system with random delays. It is possible to conclude that presented aproach provides an elegant and conceptually simple solution of simulation problems related to hybrid systems. The aggregate based approach is applicable for construction of general models describing probabilistic interactions between events and dynamics. This approach clearly allows simulation of dynamics and systematic transitions considering stimuli and delays.

References

1. J. Devooght, Dynamic reliability, *Advances in Nuclear Sciences and Technology*, **25**, 215–278 (1997).
2. P.E. Labeau, C. Smidts, S. Swaminathan, Dynamic reliability: towards an integrated platform for probabilistic risk assessment, *Reliability Engineering and System Safety*, **68**, 219–254 (2000).
3. P.E. Labeau, J.M. Izquierdo, Modeling PSA problems-I: the stimulus-driven theory of probabilistic dynamics, *Nuclear Science and Engineering*, **150**, 1–25 (2005).
4. R. Alzbutas, V. Janilionis, The simulation of dynamic systems using combined modelling, *Mathematical Modelling and Analysis*, **5**, 7–17 (2000).
5. R. Alzbutas, V. Janilionis, Integration of analytical and Monte Carlo methods for dynamic system safety analysis, advances in safety and reliability, in: *Proc. of European Safety & Reliability Conference* (ESREL'05), Tri city, Poland, Taylor & Francis Group (2005), pp. 61–67.
6. R. Alzbutas, V. Janilionis, Dynamic systems simulation using APL2, in: *Proceedings of the APL'99 International Conference*, (1999), pp. 20–25.
7. H. Pranevičius, The use of PLA for creation of simulation models, in: *Proceedings of the 2004 Summer Computer Simulation Conference* (SCSC 2004), San Jose, California, USA (2004), pp. 179–184.
8. H. Pranevičius, Formal specification and analysis of distributed systems, *Journal of Intelligent Manufacturing*, **9**, 559–569 (1998).

REZIUMĖ

R. Alzbutas, V. Janilionis. Stimuliuojama dinamika ir jos modeliavimas

Darbe nagrinėjami tikimybinės dinamikos bei dinaminio patikimumo veiksniai susieti su stimuliuojamos dinamikos apibrėžimu ir agregatinio modeliavimo metodo taikymu, analitinis ir imitacinis hibridinių sistemų formalizavimo ir modeliavimo būdai bei dinaminių sistemų modeliavimas atsižvelgiant į atsitiktines laiko trukmes tarp stimulų atsiradimo ir dinamikos kitimo. Pateikta nauja metodika, skirta tolydžių procesų ir nuo jų priklausančių įvykių sąveikos modeliavimui.