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Decision procedure for first-order linear temporal logic with semi-periodic kernels

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1. Introduction

First-order linear temporal logic (FTL, in short) is a very expressive language. Unfortunately, FTL is incomplete, in general, but it becomes complete after adding an ω-type rule. Some fragments of the first-order linear temporal logic are finitary complete and/or decidable. Here decision procedures for some fragments of FTL (with $\bigcirc$(Next) and $\Box$(Always)) are presented.

2. Decision procedure for first-order linear temporal logic with periodic kernels

In order to describe a decision procedure for first-order linear temporal logic with so-called semi-periodic kernels at first we present a modification of the decision procedure for FTL fragment with periodic kernels [1, 2, 3, 4].

DEFINITION 1 (kernel formula and CH-sequent, non-repeating and periodic conditions, induction-free CH-sequent). Let $\bar{x} = x_1, \ldots, x_n$ $(n \geqslant 1)$; x_i $(1 \leqslant i \leqslant n)$ is a variable; $\bar{b} = b_1, \ldots, b_k$ (b_j $(1 \leqslant j \leqslant k)$ is a constant). Let $m = n - k$ and $\bar{x}_1 = x_1, \ldots, x_m$ $m \leqslant n$. Then $\bar{x}_1\bar{b} = x_1, \ldots, x_m, b_1, \ldots, b_k$. A formula $\Box B$ is a kernel formula, if $B = \forall\bar{x}(E(\bar{x}) \supset P^1(\bar{x}_1\bar{b}))$, where $P^1(\bar{x}_1\bar{b})$ means $\bigcirc P(\bar{x}_1\bar{b})$. A sequent S is a CH-sequent if $S = \Sigma, \Box\Omega \rightarrow \Box^{\circ}A$, where $\Sigma = \varnothing$ or consists of atomic formulas, $\Box\Omega$ consists of kernel formulas; $A = \bigvee_{i=1}^{m} \exists\bar{y}E_i^{k_i}(\bar{y})$, $\forall i(k_i \geqslant 0)$, $\bar{y} = y_1, \ldots, y_j$, in a separate case, y_i is a constant and $\exists y_i = \varnothing$. $\Box^0 \in \{\varnothing, \Box\}$; if $\Box^0 = \varnothing$, then the CH-sequent is an induction-free. Each CH-sequent must satisfy the following conditions:

Non-repeating conditions: (a) if $P(\bar{b}) \in \Sigma$, then $P(\bar{c}) \notin \Sigma$;

(b) If for every i $\Box\forall\bar{x}_i(P_i(\bar{x}_i) \supset Q_i^1(\bar{x}_{i1}\bar{b}_i)) \in \Box\Omega$ and for every j $\Box\forall\bar{x}_j(P_j(\bar{x}_j) \supset Q_j^1(\bar{x}_{j1}\bar{b}_j)) \in \Box\Omega$ then $P_i \neq P_j$ and $Q_i \neq Q_j$.

Periodic condition:

$$\begin{aligned}\Box\Omega = {} & \Box\forall\bar{x}_1(E(\bar{x}_1) \supset E_1^1(\bar{x}_{11}\bar{b}_1)),\\ & \Box\forall\bar{x}_2(E_1(\bar{x}_2) \supset E_2^1(\bar{x}_{21}\bar{b}_2)),\\ & \quad\ldots\ldots\\ & \Box\forall\bar{x}_n(E_{n-1}(\bar{x}_n) \supset E^1(\bar{x}_{n1}\bar{b}_n)).\end{aligned}$$

DEFINITION 2 (operation (+), compatible atomic formula). Let $S = \Sigma, \Box\Omega \rightarrow \Box^0 A$ be a CH-sequent and $E(\bar{c}) \in \Sigma$. Then $(E(\bar{c}))^+ = P(\bar{c}_1\bar{b})$, if $\Box\forall\bar{x}(E(\bar{x}) \supset P^1(\bar{x}_1\bar{b})) \in \Box\Omega$ (where $\bar{x} = x_1, \ldots, x_n$ $(n > 0)$; $x_1 = x_1, \ldots, x_m$ $(m \leqslant n)$; $\bar{b} = b_1, \ldots, b_k$ $(m + k = n)$; $\bar{c} = c_1, \ldots, c_n$; $\bar{c}_1 = c_1, \ldots, c_m$). In this case the atomic formula $E(\bar{c})$ is compatible with the kernel $\Box\Omega$. Otherwise, $(E(\bar{c}))^+ = \varnothing$. Let $\Sigma = E_1, \ldots, E_n$, then $(\Sigma)^+ = (E_1)^+, \ldots, (E_n)^+$.

DEFINITION 3 (calculi CHG^*_ω, CHG^*). A calculus CHG^*_ω is defined by the following axiom:

$$\Gamma, E(\bar{b}_0, \bar{c}_0) \rightarrow \exists\bar{y}E(\bar{y}, \bar{c}_0) \vee \bigvee_{i=1}^{m} \exists\bar{y}_i E_i^{k_i}(\bar{y}_i)$$

here and below $A \vee B = B \vee A$ for arbitrary formulas A and B; the ω-type rule

$$\frac{\Gamma \rightarrow A;\ \Gamma \rightarrow A^1; \ldots; \Gamma \rightarrow A^k; \ldots}{\Gamma \rightarrow \Box A}\ (\rightarrow \Box_\omega),$$

and the the following rule:

$$\frac{(\Sigma)^+, \Box\Omega \rightarrow A^{-1}}{\Sigma, \Box\Omega \rightarrow A}\ (ISIF),$$

where A^{-1} denotes the formula which is obtained from A, replacing the atomic formula $E_i^{k_i}(\bar{y})$ by $E_i^{k_i-1}(\bar{y})$, moreover, if $k_i - 1 < 0$ then the i-th disjunctive component is omitted.

Theorem 1 (soundness and ω-completeness of CHG^*_ω). *Let S be a CH-sequent. Then* $\forall M \models S \iff CHG^*_\omega \vdash S$.

Proof. Analogously as in [1].

Lemma 1. *The calculus CHG^* is decidable for induction-free CH-sequents.*

Proof. Follows from the shape of the calculus CHG^*.

DEFINITION 4 (generalized integrated separation rule: (GIS)). Let $S = \Sigma, \Box\Omega \rightarrow \Box A$ be a CH-sequent. Let $(\Sigma)^+$ means the same as in Definition 2. Then the generalized separation rule (GIS) is as follows:

$$\frac{\Sigma, \Box\Omega \rightarrow A;\ (\Sigma)^+, \Box\Omega \rightarrow \Box A}{\Sigma, \Box\Omega \rightarrow \Box A}\ (GIS).$$

If the left premise of (GIS) is such that $CHG^* \vdash \Sigma, \Box\Omega \rightarrow A$, we say that the bottom-up application of (GIS) is successful; in the opposite case, the bottom-up application of (GIS) is not successful.

The notation $(GIS)(S) = S^*$ means that after a successful bottom-up application of (GIS) to a sequent S we get the sequent S^* as the right premise of (GIS).

Lemma 2. *The rule (GIS) is admissible and invertible in CHG^*_ω.*

Proof. Analogously as in [2].

DEFINITION 5 (saturated CH-sequent). Let $\Sigma, \Box\Omega \rightarrow \Box A$ be a CH-sequent and $E(\bar{c}\bar{b}) \in \Sigma$. Let us define the rank of $E(\bar{c}\bar{b})$ (in symbols: $r(E(\bar{c}\bar{b}))$): $r(E(\bar{c}\bar{b})) = 0$, if $\forall \bar{x}(Q(\bar{x}) \supset E^1(\bar{x}_1\bar{b})) \in \Box\Omega$, otherwise, $r(E(\bar{c}\bar{b})) = 1$. Let $S = E_1, \ldots, E_n, \Box\Omega \rightarrow \Box A$, then $r(S) = \sum_{i=1}^{n} r(E_i)$. Let S be a CH-sequent, then S is a saturated CH-sequent if $r(S) = 0$.

Lemma 3. *Let $S = \Sigma, \Box\Omega \rightarrow \Box A$ be a CH-sequent and $r(S) > 0$. Then either $CHG^*_\omega \nvdash S$ or $CHG^*_\omega \vdash S^* = (\Sigma)^+, \Box\Omega \rightarrow \Box A$ and $r(S^*) = 0$, i.e., S^* is a saturated CH-sequent.*

Proof. Analogously as in [2].

Lemma 4. *The problem of constructing a saturated CH-sequent S^* from an arbitrary CH-sequent S is decidable.*

Proof. Follows from decidability of the calculus CHG^*.

DEFINITION 6 (procedure $Re^k(S)$, parametrical part of $Re^k(S)$). Let $S = \Sigma, \Box\Omega \rightarrow \Box A$ be a saturated CH-sequent and $|\Box\Omega|$ be the number of kernel formulas in $\Box\Omega$, denoted by $p(S)$. Thus, $p(S) = |\Box\Omega|$. Then $Re^0(S) = S$. Let $Re^k(S) = S_k = \Sigma_k, \Box\Omega \rightarrow \Box A$, then $Re^{k+1}(S)$ is defined in the following way:

1. Let us bottom-up apply the rule (GIS) to S_k and S_{k1}, S_{k2} be the left and the right premises of the application of (GIS).

2. If $CHG^* \nvdash S_{k1}$, then $Re^{k+1}(S) = \perp$ (failure) and the calculation of $Re^{k+1}(S)$ is stopped.

3. Let $CHG^* \vdash S_{k1}$ (it means that the bottom-up application of (GIS) is successful). Then $Re^{k+1}(S) = S_{k2} = (\Sigma_k)^+, \Box\Omega \rightarrow \Box A$; $(\Sigma_k)^+$ is called a parametrical part of $Re^{k+1}(S)$;

4. If $Re^{k+1}(S) = S_{k2}$ and $k+1 = |\Box\Omega|$, then the calculation of $Re^{k+1}(S)$ is finished.

The notation $Re^k(S) \neq \perp$ $(k \leqslant p(S))$ means that all the bottom-up applications of (GIS) in the calculation of $Re^k(S)$ are successful.

Lemma 5. *The problem of calculation of $Re^k(S)$ is decidable.*

Proof. Follows from decidability of the calculus CHG^*.

Lemma 6 (loop property). *Let S be a saturated CH-sequent and $Re^k(S) \neq \perp$ $(k \leqslant p(S))$. Then $Re^l(S) = S$ and $(l = p(S))$.*

Proof. Analogously as in [2].

DEFINITION 7 (looping CH-sequent). Let S be a saturated CH-sequent and $Re^l(S) = S$ (where $l = p(S)$). Then S is a looping CH-sequent.

DEFINITION 8 (saturation calculus $CHSat$, CH-sequent derivable in $CHSat$). The calculus $CHSat$ consists of a preliminary step (by means of which a saturated CH-sequent is generated) and procedure $Re^k(S)$. A CH-sequent S is derivable in $CHSat$ (in symbols: $CHSat \vdash S$) if (1) $(GIS)(S) = S^*$, where S^* is a saturated CH-sequent; if $r(S) = 0$, i.e., if S is a saturated CH-sequent, then $S = S^*$; (2) $Re^l(S^*) = S^*$, where $l = p(S^*)$; otherwise, $CHSat \nvdash S$.

From Lemmas 4 and 5 we get

Theorem 2. *A calculus $CHSat$ is decidable for the class of CH-sequents.*

The saturation calculus $CHSat$ is justified using a so-called invariant calculus $CHIN$.

DEFINITION 9 (calculus CHG^+). A calculus CHG^+ is obtained from the calculus CHG^* by adding

1) the axiom $\Gamma, \Box A \to \Delta, \Box A^1$,

2) the traditional invertible logical rules $(\wedge \to)$, $(\vee \to)$, $(\to \wedge)$, $(\to \vee)$,

3) by modifying the axiom of CHG^* by adding a multiset Δ in the succedent of the axiom.

DEFINITION 10 (invariant calculus $CHIN$). A invariant calculus $CHIN$ is obtained from the calculus CHG^+ by adding the following rule:

$$\frac{\Sigma, \Box\Omega \to I;\ I \to I^1;\ I \to A}{\Sigma, \Box\Omega \to \Box A}\ (\to \Box)$$

The rule $(\to \Box)$ satisfies the following conditions:

(1) the conclusion of $(\to \Box)$, i.e., the sequent $S = \Sigma, \Box\Omega \to \Box A$ is such that $Re^l(S) = S$, $l = p(S)$.

(2) $I = \bigvee_{i=1}^{p} \Sigma_i^\wedge \wedge (\Box\Omega)^\wedge$, where Σ_i is the parametrical part of $Re^k(S)$, $k \in \{1, 2, \ldots, p\}$, $p = |\Box\Omega|$; $\Sigma_i^\wedge$ is the conjunction of formulas from Σ_i.

Theorem 3. *Let S be a CH-sequent. Then $CHSat \vdash S \iff CHIN \vdash S \iff CHG^*_\omega \vdash S$.*

Proof. Analogously as in [3].

From Theorems 1 and 3 we get

Theorem 4 (soundness and ω-completeness of the calculi $CHSat$ and $CHIN$). *$\forall M \models S \iff I \vdash S$, where $I \in \{CHSat, CHIN\}$, S is a CH-sequent.*

3. Decision procedure for first-order linear temporal logic with semi-periodic kernels

Now we extend the saturation calculus $CHSat$ for semi-periodic CH-sequents.

DEFINITION 11 (strictly non-periodic kernel, connected kernel, strictly non-periodic CH-sequent). A kernel $\Box\Omega$ is strictly non-periodic if any subset of $\Box\Omega$ does not satisfy the periodic condition (see Definition 1). A kernel $\Box\Omega$ is connected if $\Box\Omega = \Box\forall\bar{x}_1(E_1(\bar{x}_1) \supset E_2^1(\bar{x}_{11}\bar{b}_1)), \Box\forall\bar{x}_2(E_2(\bar{x}_2) \supset E_3^1(\bar{x}_{12}\bar{b}_{21})), \ldots, \Box\forall\bar{x}_k(E_k(\bar{x}_k) \supset E_{k+1}^1(\bar{x}_{1k}\bar{b}_k)), \Box\forall\bar{x}_{k+1}(E_{k+1}(\bar{x}_{k+1}) \supset E_{k+2}^1(\bar{x}_{1k+1}\bar{b}_{k+1})), \ldots, \Box\forall\bar{x}_n(E_n(\bar{x}_n) \supset E_{n+1}^1(\bar{x}_{1n}\bar{b}_n))$, where $k \geqslant 1$. A definition of strictly non-periodic CH-sequent is obtained from definition of CH-sequents replacing periodic condition by strictly non-periodic condition.

Lemma 7. *Let $S = \Sigma, \Box\Omega \rightarrow \Box A$ be a strictly non-periodic CH-sequent, then $CHG_\omega^* \nvdash S$, i.e. a strictly non-periodic CH-sequent $S = \Sigma, \Box\Omega \rightarrow \Box A$ is invalid.*

Proof. Using invertibility of the rule $(\rightarrow \Box_\omega)$ instead of the sequent S we can consider sequent $S_k = \Sigma, \Box\Omega \rightarrow A^k$ $(k \in \omega)$ and instead of the calculus CHG_ω^* we can consider the finitary calculus CHG^*. Lemma is proved using induction on $|\Sigma|$.

DEFINITION 12 (multi-periodic CH-sequent). A sequent S is a multi-periodic CH-sequent, if $S = \Sigma, \Box\Omega_1, \ldots, \Box\Omega_n \rightarrow \Box^\circ A$, where Σ, $\Box\Omega_i$ $(1 \leqslant i \leqslant n)$, $\Box^\circ$, A are the same as in definition of CH-sequents, see Definition 1, i.e., each kernel $\Box\Omega_i$ $(1 \leqslant i \leqslant n)$ satisfies the periodic condition; if $n = 1$ then multi-periodic CH-sequents coincides with CH-sequent.

DEFINITION 13 (semi-periodic CH-sequent). A sequent S is a semi-periodic CH-sequent if $S = \Sigma, \Box\Omega, \Box\Omega_1, \ldots, \Box\Omega_n \rightarrow \Box^\circ A$, where Σ, $\Box\Omega_i$ $(1 \leqslant i \leqslant n)$; $\Box^\circ$, A are the same as in the case of multi-periodic sequent and $\Box\Omega$ satisfies strictly non-periodic condition; the kernel $\Box\Omega$ is non-periodic part and $\Box\Omega_1, \ldots, \Box\Omega_n$ is multi-periodic part of semi-periodic CH-sequent.

DEFINITION 14 (rank of semi-periodic CH-sequent, saturated semi-periodic CH-sequent). Let Σ,$\Box\Omega$,$\Box\Omega_1$,...,$\Box\Omega_n \rightarrow \Box A$ be a semi-periodic CH-sequent and $E(\bar{c}\bar{b}) \in \Sigma$. Let us define the rank of $E(\bar{c}\bar{b})$ (in symbols $r^*(E(\bar{c}\bar{b}))$): $r^*(E(\bar{c}\bar{b})) = 0$, if $\forall\bar{x}(Q(\bar{x}) \supset E^1(\bar{x}_1\bar{b})) \in \Box\Omega_i$ (where $\Box\Omega_i$ belongs to multi-periodic part of S), otherwise $r^*(E(\bar{c}\bar{b})) = 1$. Let $S = E_1, \ldots, E_n, \Box\Omega \rightarrow \Box A$, then $r^*(S) = \sum_{i=1}^{n} r^*(E_i)$. Let S be a semi-periodic CH-sequent, then S is a saturated semi-periodic CH-sequent, if $r^*(S) = 0$.

Now we define a preliminary procedure for proposed saturation-based procedure $CHSat^*$ for semi-periodic CH-sequents. This preliminary procedure will be called the preliminary k-th resolvent (in symbols: $P^*Re^k(S)$). The aim of $P^*Re^k(S)$ is to generate (from a given semi-periodic CH-sequent S) a saturated semi-periodic CH-sequent S^*.

DEFINITION 15 (preliminary k-th resolvent: $P^*Re^k(S)$). Let S be a semi-periodic CH-sequent, then $P^*Re^0(S) = S$. If $r^*(S) = 0$ then calculation of $P^*Re^k(S)$ is finished. Let $r^*(S) > 0$ and $P^*Re^k(S) = S_k = \Sigma, \Box\Omega \to \Box A$, then $P^*Re^{k+1}(S)$ is defined as follows:

1. Let us bottom-up apply the rule (GIS) to S_k, and S_{k1}, S_{k2} be the left and right premises of the application of (GIS).
2. If $CHG^* \nvdash S_{k1}$, then $P^*Re^{k+1}(S) = \bot$ (false) and calculation of $P^*Re^{k+1}(S)$ is stopped.
3. Let $CHG^* \vdash S_{k1}$, then $P^*Re^{k+1}(S) = S_{k2} = (\Sigma)^+, \Box\Omega \to \Box A$.
4. If $P^*Re^{k+1}(S) = S_{k2}$ and $r^*(S) = 0$ then calculation of $P^*Re^{k+1}(S)$ is finished.

Notation $P^*Re^k(S \neq \bot)$ $(k \in \omega)$ means that all bottom-up applications of (GIS) in calculation of $P^*Re^k(S)$ are successful.

Now we establish a simple upper bound of steps in calculation of $P^*Re^k(S)$.

Lemma 8. *Let $S = \Sigma, \Box\Omega, \Box\Omega_1, \ldots, \Box\Omega_p \to \Box A$ be a semi-periodic CH-sequent, where $\Box\Omega$ is non-periodic part of S. Let $P^*Re^k(S) \neq \bot$ ($k \leqslant n+1$, where $n = |\Box\Omega|$). Then $\exists i\, (P^*Re^i(S) = S^*)$ such that $i \leqslant n+1$ and $r(S^*) = 0$.*

Proof. If $r(S) = 0$, then $S = S^*$. Let $r(S) > 0$. Let $\Box\Omega = \Box\forall\bar{x}_1(E_1^*(\bar{x}_1) \supset E_2^1(\bar{x}_{11}\bar{b}_1))$, $\Box\forall\bar{x}_2(E_2^*(\bar{x}_2) \supset E_3^1(\bar{x}_{21}\bar{b}_2))$, …, $\Box\forall\bar{x}_n(E_n^*(\bar{x}_n) \supset E_{n+1}^1(\bar{x}_{n1}\bar{b}_n))$. Let $\Sigma = \Sigma_1, \Sigma_2$, where Σ_1 consists of atomic formulas which are compatible with strictly non-periodic kernel $\Box\Omega$, Σ_2 consists of atomic formulas which are compatible with multi-periodic kernel $\Box\Omega_1$, …, $\Box\Omega_p$. Without loss of generality we can assume that $\Sigma_1 = E_1^*(\bar{c}_1)$, …, $E_i^*(\bar{c}_i)$, $E_{i+1}^*(\bar{c}_{i+1})$, …, $E_l^*(\bar{c}_l)$ $(l \leqslant n)$. Since $r(S) > 0$, $l > 0$. Let us consider two cases.

1. $\Box\Omega$ is a connected kernel, i.e., $E_2 = E_2^*$; …; $E_i = E_i^*$; $E_{i+1} \neq E_{i+1}^*$; …; $E_l \neq E_l^*$, where $0 \leqslant i \leqslant l \leqslant n$. In this case (using that $E_{i+1} \neq E_{i+1}^*$) calculating $i+1$-time $(i \leqslant l \leqslant n)$ $P^*Re^k(S)$ we get that $P^*Re^{i+1}(S) = S^* = \Sigma_2^*, \Box\Omega \to \Box A$, where Σ_2^* consists of atomic formulas which are compatible with multi-periodic kernel $\Box\Omega_1, \ldots, \Box\Omega_p$. Therefore after $i+1$-steps $(i \leqslant n)$ we get saturated multi-periodic sequent.

2. $\Box\Omega$ is not connected kernel. In this case we get that $P^*Re^i(S) = S^*$, where $r^*(S) = 0$ and $i = 1$ or $i = 2$.

Lemma 9. *For a semi-periodic CH-sequent S, the problem of generation of the saturated semi-periodic CH-sequent is decidable.*

Proof. Follows from decidability of condition of the successful bottom-up application of (GIS) and Lemma 8.

Now we define the basic part of $CHSat^*$ – the modified k_i-th resolvent (in short: $R^*e_i^k(S)$).

First we define a modified generalized integrated separation rule, a rule (GIS_i^*).

DEFINITION 16 (modified generalized integrated separation rule: (GIS_i^*), successful application of (GIS_i^*)). Let S be a saturated semi-periodic CH-sequent, and $S = \Sigma_1, \ldots, \Sigma_l, \Box\Omega, \Box\Omega_1, \ldots, \Box\Omega_l, \ldots, \Box\Omega_p \to \Box A$, where $\Box\Omega$ is a non-periodic part of S; $\Box\Omega_1, \ldots, \Box\Omega_p$ is a periodic part of S; the kernel formulas from $\Box\Omega_i$ $(1 \leqslant i \leqslant l)$ are compatible with parametrical formulas from Σ_i. Then the rule (GIS_i^*) has the following shape:

$$\frac{\Sigma_i, \Box\Omega_i \to A; \ (\Sigma_i)^+, \Box\Omega_i \to \Box A}{\Sigma_1, \ldots, \Sigma_l, \Box\Omega^* \to \Box A} \ (GIS_i^*),$$

where $i \in \{1, \ldots, l\}$, $(\Sigma_i)^+$ means the same as in Definition 1, $\Box\Omega^* = \Box\Omega, \Box\Omega_1, \ldots, \Box\Omega_p$. If the left premise of (GIS_i^*), i.e., the sequent $S_i = \Sigma_i, \Box\Omega_i \to A$ such that $CHG^* \vdash S_i$ we say that bottom-up application of (GIS_i^*) is successful.

Lemma 10. *(a) The rule (GIS_i^*) is admissible in CHG_ω^*.*

(b) The rule (GIS_i^*) is existential invertible in CHG_ω^*, i.e., if $CHG_\omega^* \vdash \Sigma_1, \ldots, \Sigma_l, \Box\Omega_i^* \to \Box A$, then $\exists i$ such $CHG^* \vdash \Sigma_i, \Box\Omega_i \to A$ and $CHG_\omega^* \vdash (\Sigma_i)^+, \Box\Omega_i \to \Box A$.

Proof. Analogously as in [2].

DEFINITION 17 (procedure $Re_i^k(S)$, parametrical part of $Re_i^k(S)$). Let $S = \Sigma_1, \ldots, \Sigma_l, \Box\Omega, \Box\Omega_1, \ldots, \Box\Omega_l \to \Box A$ be a saturated and semi-periodic CH-sequent, where $\Box\Omega$ is a non-periodic part of S and $\Box\Omega_1, \ldots, \Box\Omega_l$ is a periodic part of S. Then the $Re_i^k(S)$) is defined in the following way: $\forall i$ $(1 \leqslant i \leqslant l)$ $Re_i^0(S) = S$.

$Re_i^1(S)$ is defined by means of the following steps:

1. $i := 1$.

2. Let us apply the rule (GIS_i) to S and S_{1i}, S_{2i} be the left and the right premises of (GIS_i).

3. If $CHG^* \nvdash S_{1i}$, then $Re_i^1(S) = \bot$ (failure); if $i < l$, then $i := i + 1$, go to Step 2 else stop.

4. Let $CHG^* \vdash S_{1i}$ (it means that bottom-up application of (GIS_i) is successful). Then $Re_i^1(S) = S_{1i} = (\Sigma_i)^+, \Box\Omega_i \to \Box A$; $(\Sigma_i)^+$ is called a parametrical part of $Re_i^1(S)$.

Let $Re_i^k(S) = S_i^k = \Sigma_i^*, \Box\Omega_i \to \Box A$, then $Re_i^{k+1}(S)$ is defined by means of the following steps:

1. Let us bottom-up apply the rule (GIS) to S_i^k and $S_{i_1}^k, S_{i_2}^k$ be the left and the right premises of the application of (GIS).

2. If $CHG^* \nvdash S_{i_1}^k$, then $Re_i^{k+1}(S) = \bot$ (failure) and the calculation of $Re_i^{k+1}(S)$ is stopped.

3. Let $CHG^* \vdash S^k_{i_1}$ (it means that the bottom-up application of (GIS) is successful). Then $Re^{k+1}_i(S) = S^k_{i_2} = (\Sigma^*_i)^+, \Box\Omega_i \to \Box A$; $(\Sigma^*_i)^+$ will be called a parametrical part of $Re^{k+1}_i(S)$.

4. If $Re^{k+1}_i(S) = S^k_{i_2}$ and $k+1 = |\Box\Omega_i|$, then the calculation of $Re^{k+1}_i(S)$ is finished.

The notation $Re^k_i(S) \neq\bot$ $(k \leqslant |\Box\Omega_i|)$ means that the bottom-up application of (GIS_i) in the calculation of $Re^1_i(S)$ and all the bottom-up applications of (GIS) in the calculation of $Re^k_i(S)$ $(1 < k \leqslant |\Box\Omega_i|)$ are successful.

Lemma 11. *The problem of calculation of $Re^k_i(S)$ is decidable.*

Proof. Follows from decidability of CHG^* and definition of $Re^k_i(S)$.

Lemma 12 (loop property). *Let $S = \Sigma_1, \ldots, \Sigma_l, \Box\Omega, \Box\Omega_1, \ldots, \Box\Omega_l \to \Box A$ be a saturated and semi-periodic CH-sequent. Let $Re^k_i(S) \neq\bot$ $(k \leqslant |\Box\Omega_i|)$. Then $\exists i$ such that $Re^q_i(S) = S^*_i = \Sigma^*_i, \Box\Omega_i \to \Box A$, where $q = |\Box\Omega_i|$ and S^*_i is the looping CH-sequent.*

Proof. Analogously as in [2].

DEFINITION 18 (saturation calculus $CHSat^*$). The saturation calculus $CHSat^*$ consists of two decidable procedures:

(1) $PRe^k(S)$, which generates a saturated and semi-periodic CH-sequent S^*, and
(2) $Re^k_i(S^*)$, which generates a looping CH-sequent S^{**}.

From Lemmas 9 and 11 we get

Theorem 5. *The saturation calculus $CHSat^*$ is decidable for the class of semi-periodic CH-sequents.*

DEFINITION 19 (invariant calculus $CHIN^*$). An invariant calculus $CHIN^*$ is obtained from the invariant calculus $CHIN$, replacing parametrical parts of the procedure $Re^k(S)$ by parametrical parts of the procedure $Re^k_i(S)$.

Theorem 6 (soundness and completeness of the calculi $CHSat^*$ and $CHIN^*$). *Let S be a semi-periodic CH-sequent. Then $\forall M \models S \iff I \vdash S$, where $I \in \{CHSat^*, CHIN^*\}$.*

Proof. Analogously as in Theorem 4.

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Išsprendžiamoji procedūra kvantorinės tiesinio laiko logikos fragmentui su pusiau-periodiniais branduoliais

R. Pliuškevičius

Pateikiamas apibendrinimas ankstesnių autoriaus darbų apie išsprendžiamąją procedūrą kvantorinės tiesinio laiko logikos fragmentui su periodiniais branduoliais. Remiantis šiais rezultatais pateikiama išsprendžiamoji procedūra minėtos logikos fragmentui su pusiau-periodiniais branduoliais.