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Parallel computer algebra system

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1. Preface

VIBRAN is the computer algebra system (VIBRotechnika and ANalitika) designed for implementation of operations with analytical expressions.

The first version of VIBRAN was developed in 1979 for IBM/360. A later version, VIBRAN Compact, was designed in 1981 for PDP/11. The same year was published an improvement and it was running on IBM PC. Later, in 1990, was developed Unix family version (AIX, HP-UX). Finally, in 1998 the PARAVIB for IBM SP/2 was designed. PARAVIB is the parallel version of the computer algebra system VIBRAN.

Computer algebra system VIBRAN is written in FORTRAN. VIBRAN consists of pre-processor and complex of subprograms for analytical or service transformations of expressions such as polynomials, rational functions, trigonometric series and sparse matrices.

All variables and their degrees in expression are stored separately. Such storage allows analytical operations to be changed with operations with matrices.

The latest version of VIBRAN is PARAVIB, the parallel VIBRAN version. The computer algebra system PARAVIB is a HPF (High Performance Fortran) pre-processor for analytical perturbation with very long polynomials, rational functions and trigonometric series. PARAVIB consists of pre-processor and procedures library. Using pre-processor it is possible define analytical objects and to design new procedures that can be included into the PARAVIB library.

The VIBRAN applications are widely used in many fields. For instance, in generation of equations of motion for multi-degree-of-freedom non-linear mechanical systems. Equations of motion are separated into the linear and non-linear parts. This method provides smaller expressions for analytical computation and allows analyze systems with greater order. Second utilization of VIBRAN is numeric-symbolic analysis of the non-linear oscillation systems. Harmonic balance, small parameter and asymptotic methods are implemented for non-linear oscillation systems with multi-degree-of-freedom analysis. The computer algebra technologies also are used for the Finite element method, specifically for formulation of finite elements. These technologies give big reduction of required computer resources for structural matrices. In the control system theory analytical perturbations with VIBRAN programs avoid complex terms from numerical calculations. Usage of VIBRAN in the real-time dynamic models of industrial robots saves a great number of floating point multiplications and allows intelligent control schemes realization, using parallel processors for separate motion components.

2. VIBRAN variables representation in computer memory

All analytical expressions such as variables, polynomials, constants are VIBRAN variables, all FORTRAN data types can be used too. System variables can be represented in packed and unpacked format.

VIBRAN represents the polynomial and rational terms of analytical expressions in three different arrays: for coefficients of terms, variables, and for powers of variables. Thus, all analytical operations are replaced by matrix arithmetic on those tree arrays. The last, biggest array for powers of variables is represented in packed, compact form with requirement of one byte memory, it can be represented in unpacked format too.

The trigonometric terms are replaced by exponential terms and are stored in the same way as all other analytical terms. Such allocation in computer memory allows to process operations for $\sin\tau$ and $\cos\tau$ with all VIBRAN commands.

The sparse matrix is the matrix that has a small percentage of not null elements. Because of the possibility to distribute large dimension matrices in RAM and to operate only with not null elements the effective solution of many tasks may by enabled with sparse matrices technology. Besides that, this technology economizes RAM and reduces computation time. For every not null matrix element the corresponding whole number is assigned. Number type is

$$l(i, j) = i \times 100000 + j,$$

here i – the line number of not null element, j – the column number of not null element. Besides that, the indication of matrix representation is used: 0 – general type matrix, 1 – symmetrical matrix.

Every matrix is stored in two arrays: 1) analytical, 2) integer (vector of control information). First element of control information vector keeps the sparse matrix dimension (the number of lines and columns in the packed format). Second element keeps the number of not null elements in the matrix. For symmetrical matrices only the elements in the first triangle are stored. The dimension of integer array must be twice bigger comparing with analytical one. The information in the control vector must correspond to not null elements of matrix.

3. VIBRAN pre-processor language syntax, commands and procedures. PARAVIB extensions

3.1. *Structure of VIBRAN program. VIBRAN program processing*

The VIBRAN program consists of commands for analytical manipulations and FORTRAN operators. The program components are the main program and one or more subprograms.

The VIBRAN program must be written by FORTRAN rules and satisfy a few special requirements: the program have to be started with VIBRAN descriptive operators for analytical expressions – POLINOM and/or RACIONAL. The description of VIBRAN variables must be clear and the names of VIBRAN and FORTRAN variables used in commands must vary. At the end of the program module the list of VIBRAN variables, their types and dimensions (if variable is an array) is issued.

Finally, the names for analytical variables and analytical expressions are written by FORTRAN language rules; their length is limited to four symbols. Other variables names depend on FORTRAN rules.

In the VIBRAN program the FORTRAN commands, SUBROUTINES and other objects can coexist together with VIBRAN statements. The pre-processor skips them in compilation step.

The volume of computers memory necessary for keeping analytical expressions, as mentioned, is adjusted by statements POLINOM and RACIONAL. The last operator takes twice bigger amount of memory for expression than the POLINOM. The volume of memory can be regulated depending on size of processed analytical expressions. Regulation can be done with VIBRAN operators $*P$ and $*R$, which redistribute the memory for all three arrays: $*P\ N\ M\ L, *R\ N\ M\ L$. Here N – maximum number of expression components, M – maximum number of expression variables, L – maximum number of elements in degrees matrix. Operator $*P$ is used to allocate memory for polynomial variables, $*R$ – for rational variables.

Program in VIBRAN is processed in two steps. At the first step the analytical computations are executed and it results are inserted into disk. At the second step the subsystem of FORTRAN programs generation forms the subprogram for numerical computations.

3.2. *Syntax of VIBRAN program. The Commands and the Procedures*

MXI(A,X) – multiplication of polynomial A by variable X.

SAVE(A(3),B) – attribute to the third element of array A the value of variable B, and so on.

The user can use standard VIBRAN commands or construct commands in FORTRAN and VIBRAN languages. In this case, user commands must have VIBRAN procedure format.

Pre-processor processes VIBRAN system commands to the call of corresponding FORTRAN subroutines. Subprograms in VIBRAN are called procedures. The first statement of procedure is

$$PROCEDURE\ name\ (formal\ parameters).$$

Here name-name of the procedure which is limited to four symbol. Any FORTRAN or VIBRAN variables, names of analytical expressions and the asterisk can be the parameters of procedure. The parameters can be identified with symbolic names which length is limited to four symbols. Procedure has the same format as FORTRAN subroutine, except that the VIBRAN procedure must have at least one parameter. The call of the procedure has format of VIBRAN command:

$$name\ (real\ parameters),$$

here name-the name of the procedure, which is described in operator PROCEDURE. For example the initial data is: $*P$ 10 4 40, $*R$ 100 20 2000. The following program is example showing main program and procedure work.

```
POLINOM A1, B
RACIONAL SUM
COMMON A1, B
ANAL (A1)
ANAL (B)
PRIN(' A1', A1)
PRIN(' A2', B)
DELS (SUM)
PRIN('SIGM', SUM)
STOP
END

PROCEDURE DELS(S)
POLINOM A, B
RACIONAL S, B1
COMMON A, B
POWR (S,A, 2)
POWR (B1, B, 2)
ADDA (S,B1)
RETURN
END
```

Here commands $ANAL(A1)$, $ANAL(B)$ in main program inputs from input steam polynomials $A1$ and B. Commands $PRIN('A1', A1)$, $PRIN('A2', B)$ prints their analytical values, command $DELS(SUM)$ calls the procedure, which calculates the sum of two polynomials each of them raised to the second power/polynomial squares. The polynomials A and B analytical values procedure gets through block $COMMON$ from the calling program. The result of addition the program gets through the parameter of procedure S.

3.3. *The operations with sparse matrices of analytical expressions*

An analytical expression occupies more space in memory then single number and the execution of analytical operation takes much more time than the execution of numerical one. So the usage of sparse matrices in analytical computation is considerably effective. Computer algebra system VIBRAN has special commands and operations for sparse matrices. The array of not null elements must be described with operator POLINOM. Commands for operations with sparse matrices may be divided:

1) service (element putting/getting in/from matrix, getting of the index of not null element);

2) input-output (writing/reading of the sparse matrices to/from the file, insertion and printing in/from files);

3) algebraic (addition, multiplication of the sparse matrices, multiplication of the sparse matrix and vector or number);

4) usage of the operator (differentiation or integration either of the whole matrix, or it element, or line, or column.

3.4. *PARAVIB extensions*

Parallel computer algebra system PARAVIB is the latest version of VIBRAN. It generates the High Perfomance FORTRAN (HPF) programs from analytical expressions. Later, these programs can be used for parallel numerical computations. Here is an example of HPF code:

```
POLINOM A(1000), B(3000)
RACIONAL C(10000)
!hpf distribute A ( block)
!hpf distribute B (cyclic)
!hpf distribute C (block) onto P
```

PARAVIB supports task parallelism, HPF local and serial routines, and FORTRAN 77 local procedures. All system components are written in FORTRAN.

4. Inferences

In the computer algebra system VIBRAN an combination of analytical and numerical calculations are implemented. The system VIBRAN performs only analytical calculations. All numerical calculations are performed using programs written high level programming languages, concretely with FORTRAN programs. For these two types of calculations VIBRAN has powerful possibilities. The first one is simply generation and output of FORTRAN like attribution statement, which can be used afterwards in FORTRAN program directly.

The second more advanced possibility in VIBRAN is special subsystem, which solves the significant problem in program generation from analytical expressions that is excess arithmetical operations. The supplied sub system generates two ready for processing optimal FORTRAN programs. The generation of programs follows in two steps: first of all, VIBRAN program performs all necessary analytical calculations and obtains analytical expressions to be used in numerical ones; secondly, VIBRAN sub system analyses these expressions, picks up all repeated multiplications and attributes these sub expressions to the elements of a special vocabulary stored in the form of FORTRAN routine. Later the second FORTAN routine is generated using the elements of vocabulary. Thus all excess multiplications may be avoided.

The specific feature of computer algebra system is the representation form of analytical expressions that are stored in matrix form. This feature distinct VIBRAN from other computer algebra systems. Analytical perturbations are changed with matrix operations. Sparse matrices technology is used for analytical operations with arrays. This technology enables effective parallel analytical perturbations. The PARAVIB system is designed for parallel numerical computational applications generation. Special PARAVIB procedure generates optimized HPF code from analytical expressions. This code can be used in the programs for parallel numerical analysis. The system picks out all repeating multiplications and stores them as an automatically generated vocabulary of additional arrays. The final expressions are generated using elements of this vocabulary and are stored in array with distributed memory.

Even if computerized symbolic manipulation is a very attractive means to perform analytic calculations with complex and long formulas and expressions, however often the combination of analytical and numerical computations is the best way for large-scale computational problems. This allows the analytic work to be pushed further before numerical computations start.

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Paralelinė kompiuterinės algebros sistema

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VIBRANAS tai kompiuterinės algebros sistema (VIBRotechnika and ANa-litika) skirta operacijoms su analitinėmis išraiškomis. Ši sistema yra parašyta su FOTRANU ir susideda iš preprocesoriaus ir paprogramių rinkinio. Analitinės išraiškos kintamieji ir jų laipsniai saugomi atskirai. Toks saugojimo būdas leidžia analitinius skaičiavimus pakeisti matricinėmis operacijomis. PARAVIBAS tai paralelinė VIBRANO versija. Ši paralelinė sistema yra HPF (High Performance Fortran) preprocesorius skirtas analitiniams skaičiavimams su labai ilgais polinomais, racionalinėmis funkcijomis, trigonometrinėmis eilutėmis bei išretintomis matricomis. PARAVIBAS susideda iš preprocesoriaus ir procedūrų bibliotekos bei palaiko paralelizmą, HPF paprogrames ir FORTRANO 77 procedūras. Išretintų matricų technologija yra naudojama analitinėse operacijose su masyvais. Naudojantis šia technologija atliekami efektyvūs paraleliniai skaičiavimai. PARAVIBE yra speciali procedūra skirta optimizuoto HPF kodo generavimui iš analitinių išraiškų. Šis kodas vėliau gali būti panaudotas paraleliniuose skaitmeniniuose skaičiavimuose. Kita galinga priemonė HPF kodo generavimui yra pasikartojančių skaičiavimų išrinkimas ir saugojimas tam tikrame žodyne. Galutinės išraiškos yra generuojamos panaudojant šį žodyną ir išsaugomos masyve su paskirstyta atmintimi. Nors kompiuterizuotos simbolinės operacijos yra labai patraukli priemonė analitinių skaičiavimų su kompleksinėmis ir ilgomis formulėmis atlikimui, tačiau dažnai analitinių ir skaitmeninių skaičiavimų kombinacija yra geresnis būdas aukštos skalės operacijų atlikimui. Tokia kombinacija leidžia atlikti analitinius veiksmus pirma skaitmeninių. VIBRANAS yra naudojamas įvairiose srityse. Pavyzdžiui netiesinėse mechaninėse sistemose judėjimo lygčių su daugeliu laisvės laipsniu generavime, taip pat baigtinių elementų formulavime ir kt.