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Models of vehicle motion

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The task is to calculate the path for a vehicle following a wire in the floor. Fig. 1 and Fig. 2 describe a vehicle with a fixed rear axle and a steering wheel. The steering wheel is not necessarily positioned on the centre line. Point P_1 is fixed point anywhere on the vehicle. Point P_2 is a point on line D, which is turning synchronously with the steering wheel. The vehicle is following a trajectory with the point P_1 (Fig. 1, task 1) or with the point P_2 (Fig. 2, task 2).

Solution of task 1

(a, b) – the coordinates of the point P_1 relative to the point R_1 $(a > 0)$. $(X(t), Y(t), \alpha(t))$ – the calculated coordinates of the point R_1 (t means time) in the global coordinate system, where $\alpha(t)$ – an angle between the centre line C of the vechile and X-axis.

The motion of the vehicle is the continual rotation about the instant center $O(t)$, which is defined by the intersection of two perpendiculars $OP_1(t)$ (the perpendicular to the trajectory line) and $OR_1(t)$ (the perpendicular to the centre line of the vehicle) (Fig. 1).

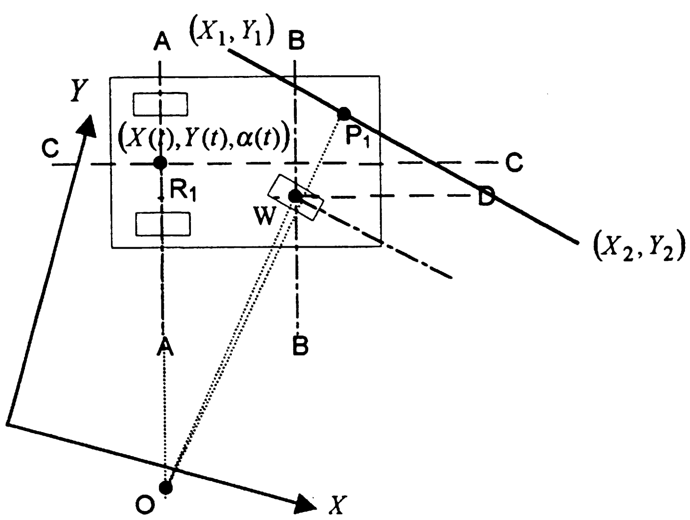

Fig. 1. The motion of the vehicle is the continual rotation about the instant center $O(t)$.

The instant turn radius of the point P_1 is $|OP_1(t)|$. The instant turn radius of the point R_1 is $|OR_1(t)|$.

The direction of rotation of the vehicle is determined by vector product

$$\overrightarrow{OP_1}(t) \times \overrightarrow{OR_1}(t) = (0, 0, \nu(t)),$$

$$m(t) = -\frac{\nu(t)}{|\nu(t)|}.$$

$m(t) = 1$, if the vehicle turns counter-clockwise and $m(t) = -1$, if the vehicle turns clockwise.

The system of differential equations, which determines the motion of the vehicle, is such:

$$\begin{cases} \dfrac{dX}{dt} = \dfrac{|OR_1(t)|}{|OP_1(t)|} \cdot \cos\alpha(t), \\ \dfrac{dY}{dt} = \dfrac{|OR_1(t)|}{|OP_1(t)|} \cdot \sin\alpha(t), \\ \dfrac{d\alpha}{dt} = \dfrac{m(t)}{|OP_1(t)|}. \end{cases}$$

There we distinguish two cases:

a) The point P_1 is on the straight line.

(X_1, Y_1) – the start point of the examined straight line.

(X_2, Y_2) – the end point of the examined straight line and the start point of the next line/arc.

$$k = \frac{X_2 - X_1}{Y_2 - Y_1}.$$

The detailed system of the differential equations gains the form

$$\begin{cases} \dfrac{dX}{dt} = \dfrac{|(a - b\cdot k)\cdot \sin\alpha(t) + (b + a\cdot k)\cdot \cos\alpha(t)|}{a\cdot \sqrt{1+k^2}} \cdot \cos\alpha(t), \\ \dfrac{dY}{dt} = \dfrac{|(a - b\cdot k)\cdot \sin\alpha(t) + (b + a\cdot k)\cdot \cos\alpha(t)|}{a\cdot \sqrt{1+k^2}} \cdot \sin\alpha(t), \\ \dfrac{d\alpha}{dt} = \dfrac{(a - b\cdot k)\cdot \sin\alpha(t) + (b + a\cdot k)\cdot \cos\alpha(t)}{|(a - b\cdot k)\cdot \sin\alpha(t) + (b + a\cdot k)\cdot \cos\alpha(t)|} \cdot \dfrac{\cos\alpha(t) - \sin\alpha(t)}{a\cdot \sqrt{1+k^2}}. \end{cases}$$

The condition

$$((a - b\cdot k)\cdot \sin\alpha(t) + (b + a\cdot k)\cdot \cos\alpha(t)) \cdot (Y_2 - Y_1) \geqslant 0$$

has to be satisfied in the initial point of the straight line.

b) The point P_1 is on the arc.

(X_1, Y_1) – the start point of the examined arc and C is the curvature of the arc.

(X_2, Y_2) – the end point of the examined arc line and the start point of the next line/arc.

The angular coefficient of the perpendicular $OP_1(t)$

$$k(t) = \tan(C\cdot t + t_1),$$

where t_1 – the initial angle of the trajectory radius in the global coordinate system.

The detailed system of the differential equations gains the form

$$\begin{cases} \dfrac{dX}{dt} = \dfrac{|(a + b\cdot k(t))\cdot\sin\alpha(t) + (b - a\cdot k(t))\cdot\cos\alpha(t)|}{a\cdot\sqrt{1 + k(t)^2}}\cdot\cos\alpha(t), \\ \dfrac{dY}{dt} = \dfrac{|(a + b\cdot k(t))\cdot\sin\alpha(t) + (b - a\cdot k(t))\cdot\cos\alpha(t)|}{a\cdot\sqrt{1 + k(t)^2}}\cdot\sin\alpha(t), \\ \dfrac{d\alpha}{dt} = \dfrac{(a + b\cdot k(t))\cdot\sin\alpha(t) + (b - a\cdot k(t))\cdot\cos\alpha(t)}{|(a + b\cdot k(t))\cdot\sin\alpha(t) + (b - a\cdot k(t))\cdot\cos\alpha(t)|}\cdot\dfrac{\cos\alpha(t) + k(t)\cdot\sin\alpha(t)}{a\cdot\sqrt{1 + k(t)^2}}. \end{cases}$$

The condition

$$((a + b\cdot k(t))\cdot\sin\alpha(t) + (b - a\cdot k(t))\cdot\cos\alpha(t))\cdot(Y_2 - Y_1) \geqslant 0$$

has to be satisfied in the each point of the arc.

Solution of task 2

(a, b) – the coordinates of the point P_2 relative to the point W $(a > 0)$. (c, d) – the coordinates of the wheel point W relative to the point R_1 $(c > 0)$. $\beta(t)$ – the turning angle of the steering wheel. $(X(t), Y(t), \alpha(t))$ – the calculated coordinates of the point R_1 (t means time) in the global coordinate system.

The motion of the vehicle is combination of the continual rotation of the vehicle about the instant center $O(t)$ and the continual rotation of system wheel-antenna about the instant center $O_1(t)$ (Fig. 2). The rotation center $O(t)$ is defined by the intersection of two perpendiculars $OR_1(t)$ (the perpendicular to the centre line of the vehicle) and $OW(t)$ (the perpendicular to the line D which is turning synchronously with the steering wheel). The rotation center $O_1(t)$ is defined by the intersection of two perpendiculars $O_1P_2(t)$ (the perpendicular to the trajectory line) and $OW(t)$.

The instant turn radius of the point P_2 is $|OP_2(t)|$. The instant turn radius of the point R_1 is $|OR_1(t)|$. The instant turn radii of the point W are $|OW(t)|$ and $|O_1W(t)|$.

The direction of rotation of the system wheel-antenna is determined by vector product

$$\overrightarrow{O_1W}(t) \times \overrightarrow{O_1P_2}(t) = (0, 0, \nu 1(t)),$$

$$m1(t) = -\frac{\nu 1(t)}{|\nu 1(t)|}.$$

$m1(t) = 1$, if the system wheel-antenna turns counter-clockwise and $m1(t) = -1$, if the system wheel-antenna turns clockwise.

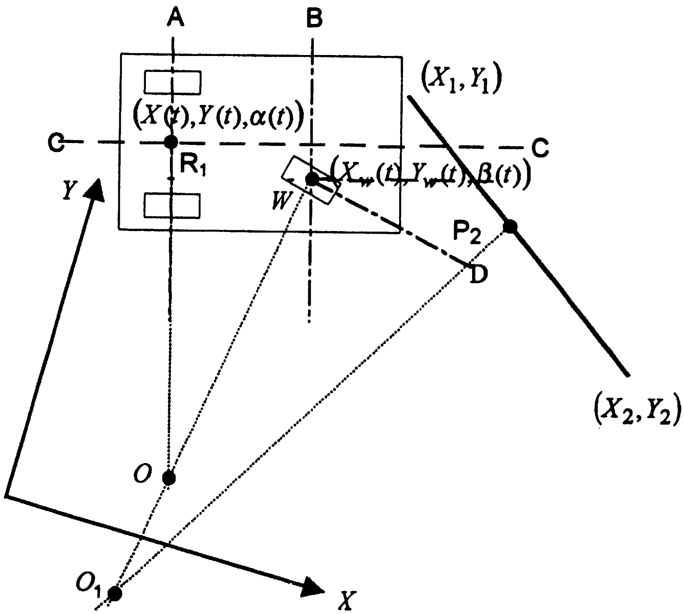

Fig. 2. The motion of the vehicle is combination of the continual rotation about the instant center $O(t)$ and $O_1(t)$.

The direction of rotation of the vehicle is determined by vector product

$$\overrightarrow{OW}(t) \times \overrightarrow{OR_1}(t) = (0, 0, \nu 2(t)),$$

$$m2(t) = -\frac{\nu 2(t)}{|\nu 2(t)|}.$$

$m2(t) = 1$, if the vehicle turns counter-clockwise and $m2(t) = -1$, if the vehicle turns clockwise.

The system of differential equations, which determines the motion of the vehicle, is such:

$$\begin{cases} \dfrac{dX}{dt} = \dfrac{|O_1W(t)|}{|O_1P_2(t)|} \cdot \dfrac{|OR_1(t)|}{|OW(t)|} \cdot \cos\alpha(t), \\ \dfrac{dY}{dt} = \dfrac{|O_1W(t)|}{|O_1P_2(t)|} \cdot \dfrac{|OR_1(t)|}{|OW(t)|} \cdot \sin\alpha(t), \\ \dfrac{d\alpha}{dt} = \dfrac{|O_1W(t)|}{|O_1P_2(t)|} \cdot \dfrac{m2(t)}{|OW(t)|}, \\ \dfrac{d\beta}{dt} = \dfrac{m1(t)}{|O_1P_2(t)|} - \dfrac{|O_1W(t)|}{|O_1P_2(t)|} \cdot \dfrac{m2(t)}{|OW(t)|}. \end{cases}$$

There we distinguish two cases:

a) The point P_2 is on the straight line.

(X_1, Y_1) – the start point of the examined straight line. (X_2, Y_2) – the end point of the examined straight line and the start point of the next line/arc.

$$k = \frac{X_2 - X_1}{Y_2 - Y_1}.$$

The detailed system of the differential equations gains the form

$$\begin{cases} \dfrac{dX}{dt} = \dfrac{|K2(t)|}{c} \cdot \cos\alpha(t) \cdot \dfrac{K1(t)}{a \cdot \sqrt{1 + k(t)^2}}, \\ \dfrac{dY}{dt} = \dfrac{|K2(t)|}{c} \cdot \sin\alpha(t) \cdot \dfrac{K1(t)}{a \cdot \sqrt{1 + k(t)^2}}, \\ \dfrac{d\alpha}{dt} = \dfrac{K2(t) \cdot \sin\beta(t)}{c \cdot |K2(t)|} \cdot \dfrac{K1(t)}{a \cdot \sqrt{1 + k(t)^2}}, \\ \dfrac{d\beta}{dt} = \left(\dfrac{(\cos(\alpha(t) + \beta(t)) + k \cdot \sin(\alpha(t) + \beta(t)))}{K1(t)} - \dfrac{K2(t) \cdot \sin\beta(t)}{c \cdot |K2(t)|} \right) \\ \qquad \times \dfrac{K1(t)}{a \cdot \sqrt{1 + k(t)^2}}, \end{cases}$$

where

$$K1(t) = (a - b \cdot k) \cdot \sin(\alpha(t) + \beta(t)) + (b + a \cdot k) \cdot \cos(\alpha(t) + \beta(t)),$$
$$K2(t) = c \cdot \cos\beta(t) + d \cdot \sin\beta(t).$$

The conditions

$$((a - b \cdot k) \cdot \sin(\alpha(t) + \beta(t)) + (b + a \cdot k) \cdot \cos(\alpha(t) + \beta(t))) \cdot (Y_2 - Y_1) \geqslant 0,$$

$$c \cdot \cos\beta(t) + d \cdot \sin\beta(t) \geqslant 0$$

have to be satisfied in the initial point of the straight line.

c) The point P_2 is on the arc.

(X_1, Y_1) – the start point of the examined arc and C is the curvature of the arc . (X_2, Y_2) – the end point of the examined arc line and the start point of the next line/arc.

The angular coefficient of the perpendicular $OP_2(t)$ $k(t) = \tan(C \cdot t + t_1)$, where t_1 – the initial angle of the trajectory radius in the global coordinate system.

The detailed system of the differential equations gains the form

$$\begin{cases} \dfrac{dX}{dt} = \dfrac{|K2(t)|}{c} \cdot \cos\alpha(t) \cdot \dfrac{K3(t)}{a \cdot \sqrt{1+k(t)^2}}, \\ \dfrac{dY}{dt} = \dfrac{|K2(t)|}{c} \cdot \sin\alpha(t) \cdot \dfrac{K3(t)}{a \cdot \sqrt{1+k(t)^2}}, \\ \dfrac{d\alpha}{dt} = \dfrac{K2(t) \cdot \sin\beta(t)}{c \cdot |K2(t)|} \cdot \dfrac{K3(t)}{a \cdot \sqrt{1+k(t)^2}}, \\ \dfrac{d\beta}{dt} = \left(\dfrac{(k(t) \cdot \sin(\alpha(t)+\beta(t)) + \cos(\alpha(t)+\beta(t)))}{K3(t)} - \dfrac{K2(t) \cdot \sin\beta(t)}{c \cdot |K2(t)|} \right) \\ \qquad \times \dfrac{K3(t)}{a \cdot \sqrt{1+k(t)^2}}, \end{cases}$$

where

$$K2(t) = c \cdot \cos\beta(t) + d \cdot \sin\beta(t),$$
$$K3(t) = (a + b \cdot k(t)) \cdot \sin(\alpha(t)+\beta(t)) + (b - a \cdot k(t)) \cdot \cos(\alpha(t)+\beta(t)) .$$

The conditions

$$((a + b \cdot k(t)) \cdot \sin(\alpha(t)+\beta(t)) + (b - a \cdot k(t)) \cdot \cos(\alpha(t)+\beta(t)))$$
$$\times C \cdot \cos(C \cdot t + t_1) \geqslant 0,$$
$$c \cdot \cos\beta(t) + d \cdot \sin\beta(t) \geqslant 0$$

have to be satisfied in the each point of the arc.

Fourth-order Runge-Kutta adaptive method was used for solving of the obtained systems of the differential equations [1,2].

References

[1] B. Kvedaras, M. Sapagovas, *Skaičiavimo metodai*, Vilnius, Mintis (1974).
[2] F.B. Hildebrand, *Introduction to numerical analysis*, McGraw-Hill (1974).

Vežimėlio judesio modeliai

N. Listopadskis

Vežimėlio judesys aprašomas diferencialinių lygčių sistemomis, kurių sprendimui panaudotas adaptyvus ketvirtos eilės Rungės ir Kutos metodas. Sukurtos programinės priemonės šių uždavinių sprendimui.