

On hydromagnetic flow due to a rotating disk with radiation effects

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Abstract. The effect of thermal radiation on the steady laminar convective hydromagnetic flow of a viscous and electrically conducting fluid due to a rotating disk of infinite extend is studied. The fluid is subjected to an external uniform magnetic field perpendicular to the plane of the disk. The governing Navier–Stokes and Maxwell equations of the hydromagnetic fluid, together with the energy equation, are transformed into nonlinear ordinary differential equations by using the von Karman similarity transformations. The resulting nonlinear ordinary differential equations are then solved numerically subject to the transformed boundary conditions by Runge–Kutta based shooting method. Comparisons with previously published works are performed and the results are found to be in excellent agreement. Numerical and graphical results for the velocity and temperature profiles as well as the skin friction and Nusselt number are presented and discussed for various parametric conditions.

Keywords: steady MHD flow, rotating disk, heat transfer.

1 Introduction

Rotating disk flows of electrically conducting fluids have practical applications in many areas, such as rotating machinery, lubrication, oceanography, computer storage devices, viscometry and crystal growth processes etc. Also the study is interesting from the mathematical point of view. The rotating disk flow is relevant to the chemical vapor deposition reactors used for depositing thin films of optical and electrical materials on substrates in the electrochemical industry. The gas flow for the operation of such reactors can be considered as a uniform axial flow incident on a rotating disk and it is desirable that the flow close to the substrate be laminar and free from instability to ensure uniform deposition.

Von Karman's swirling viscous flow [1] is a well-documented classical problem in fluid mechanics, which has several technical and industrial applications. The original

problem raised by von Karman, which is most studied by researchers in the literature, is the viscous flow motion induced by an infinite rotating disk where the fluid far from the disk is at rest. The rotating disk issues are attacked, theoretically, numerically and experimentally, by many researchers amongst many others, such as [2–5]. However, all of these results are obtained either numerically or analytically. The work by Takeda et al. [6] explored the permeable wall conditions.

The magnetohydrodynamic (MHD) fluid flow problem of a rotating disk finds special places in several science and engineering applications, for instance, in turbomachinery, in cosmical fluid dynamics, in meteorology, in computer disk drives, in geophysical fluid dynamics, in gaseous and nuclear reactors, in MHD power generators, flow meters, pumps, and so on. The first traceable interest in MHD flow was in 1907, when Northrup [7] built a MHD pump prototype. The hydromagnetic flow due to a rotating disk was first investigated by Katukani [8]. Some interesting results on the effects of the magnetic field on the steady flow due to the rotation of a disk of infinite or finite extent was pointed out by El-Mistikawy et al. [9] and Ariel [10]. Kumar et al. [11] and Turkyilmazoglu [12] covered the effects of a uniform external magnetic field on the steady flow over a rotating disk.

Rotating disk flow along with heat transfer is one of the classical problems of fluid mechanics, which has both theoretical and practical value. Heat transfer problem for rotating disk systems under different situations was analysed by many authors [13–16]. The importance of heat transfer from a rotating body can be ascertained in cases of various types of machinery, for example computer disk drives and gas turbine rotors. Hossain et al. [17] and Maleque and Sattar [18] investigated the influence of variable properties on the physical quantities of the rotating disk problem by obtaining a self-similar solution of the Navier–Stokes equations along with the energy equation. Recently, Attia [19] investigated the steady flow over a rotating disk in porous medium with heat transfer.

During the production and working life of microelectronic heat transfer devices, an electrically conducting fluid is subject to a magnetic field (Ahn and Allen [20] and Mensing et al. [21]). In such cases the fluid experiences a Lorentz force which changes the flow velocities. This in turn affects the rate of heat transfer. By knowing these compound and complicated effects, new or improved designs in the manufacturing process can be developed (Mahmud et al. [22]).

In the above mentioned studies the radiation effect is ignored. When technological processes take place at high temperatures, thermal radiation heat transfer become very important and its effects cannot be neglected (Siegel and Howell [23]; Necati [24]). Recent developments in hypersonic flights such as missile reentry rocket combustion chambers, gas cooled nuclear reactors and power plants for inter planetary flight have focused attention of researchers on thermal radiation as a mode of energy transfer, and emphasize the need for inclusion of radiative transfer in these processes. Many studies have appeared concerning the interaction of radiative flux with thermal convection flows. For example, in the context of spacecraft technology Sutton [25] as early as 1956 suggested that for temperatures ranging from 4148.89 Celsius to 1926.67 Celsius, as encountered in rocket propulsion, thermal radiation can account for up to 25 % of the total heat transfer. Accordingly, it is of interest to examine the effect of the magnetic field on such flow due to its great importance in the application fields where thermal radiation and MHD

are correlative. The process of fusing of metals in the electrical furnace by applying a magnetic field and the process of cooling of the first wall inside the nuclear reactor containment vessel where the hot plasma is isolated from the wall by applying a magnetic field are examples of such field.

Many researchers studied the effects of radiation on convective flows. Hossain and Takhar [26] studied the effects of radiation on mixed convection flow of an optically dense viscous incompressible fluid past a heated vertical plate with uniform free stream velocity and surface temperature. Raptis and Perdikis [27] investigated the MHD free convection flow in the presence of thermal radiation, Damesh et al. [28] studied the similarity analysis of magnetic field and thermal radiation effects on forced convection flow. Duwairi [29] studied the effect of both magneto forces and thermal radiation on forced convection heat transfer from constant surface heat flux surfaces in the absence of viscous and Joule heating effects.

Keeping the above curious findings in mind, the present work is devoted to examine the influence of thermal radiation on the hydromagnetic flow over a rotating infinite nonporous disk and hence the present study considers the MHD flow of a steady, laminar and electrically conducting, viscous fluid over a rotating disk. Such a study has not appeared in the literature so far and constitutes an important addition to the area of rotating disk flow studies. The equations of continuity, momentum and energy, which govern the flow field are solved numerically using fourth order Runge–Kutta based shooting method. The effects of radiation and magnetic field on the flow problem are analysed.

2 Mathematical formulation of the problem

The steady, hydromagnetic axially symmetric, incompressible flow of an electrically-conducting fluid with heat transfer due to a rotating disk in the presence of radiation has been considered. It is assumed that the fluid is infinite in extent in the positive z -direction. Let (r, ϕ, z) be the set of cylindrical polar coordinates and let the disk rotate with constant angular velocity Ω and be placed at $z = 0$ as indicated in Fig. 1.

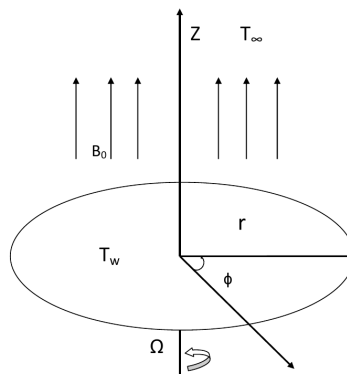


Fig. 1. Schematic diagram of the problem.

The components of the flow velocity are (u, v, w) in the directions of increasing (r, ϕ, z) respectively. The surface of the rotating disk is maintained at a uniform temperature T_w . Far away from the surface, the free stream is kept at a constant temperature T_∞ and at a constant pressure p_∞ . The fluid is assumed to be gray, emitting and absorbing heat, but not scattering medium and is assumed to be Newtonian. The external uniform magnetic field B_0 is imposed in the direction normal to the surface of the disk by considering small magnetic Reynolds number ($Re \ll 1$), it is assumed that the induced magnetic field due to the motion of the electrically conducting fluid is negligible.

Under these assumptions, the governing equations for the continuity, momentum and energy in laminar incompressible flow can be written as follows:

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial r} - \frac{v^2}{r} + w \frac{\partial u}{\partial z} + \frac{1}{\rho} \frac{\partial p}{\partial r} = \nu \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2} \right) - \frac{\sigma B_0^2}{\rho} u, \quad (2)$$

$$u \frac{\partial v}{\partial r} + \frac{uv}{r} + w \frac{\partial v}{\partial z} = \nu \left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2} + \frac{\partial^2 v}{\partial z^2} \right) - \frac{\sigma B_0^2}{\rho} v, \quad (3)$$

$$u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} + \frac{1}{\rho} \frac{\partial p}{\partial z} = \nu \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2} \right), \quad (4)$$

$$u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} = \frac{\kappa}{\rho C_p} \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right) - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial z}, \quad (5)$$

The boundary conditions for this problem are given by

$$\begin{aligned} u = 0, \quad v = \Omega r, \quad w = 0, \quad T = T_w \quad \text{at } z = 0, \\ u \rightarrow 0, \quad v \rightarrow 0, \quad T \rightarrow T_\infty, \quad p \rightarrow p_\infty \quad \text{as } z \rightarrow \infty, \end{aligned} \quad (6)$$

where κ is the thermal conductivity, $\nu = \mu/\rho$ is the kinematic viscosity of the ambient fluid, σ is the electrical conductivity, C_p is the specific heat at constant pressure, T is the temperature, ρ is the fluid density, μ is the dynamic viscosity, q_r is the radiative heat flux.

By using the Rosseland approximation for radiation for an optically thick layer one can write (see Necati [24])

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial z}, \quad (7)$$

where σ^* is the Stefan–Boltzmann constant and k^* is the mean absorption coefficient.

It is assumed that the temperature differences within the flow are such that the term T^4 may be expressed as a linear function of temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting the second and higher order terms lead to

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4, \quad (8)$$

In view of (7) and (8), (5) reduces to

$$u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} = \frac{\kappa}{\rho C_p} \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{16\sigma^* T_\infty^3}{\rho C_p 3k^*} \frac{\partial^2 T}{\partial z^2}. \quad (9)$$

3 Similarity transformation

To obtain the solutions of the governing equations, following von Karman, a dimensionless normal distance from the disk, $\eta = z(\Omega/\nu)^{1/2}$ is introduced along with the following representations for the radial, tangential and axial velocities, pressure and temperature distributions (see Shevchuk [14]):

$$\begin{aligned} u &= \Omega r F(\eta), & v &= \Omega r G(\eta), & w &= (\Omega \nu)^{\frac{1}{2}} H(\eta), \\ p - p_\infty &= 2\mu \Omega P(\eta) & \text{and} & & T - T_\infty &= \Delta T \theta(\eta), \end{aligned} \quad (10)$$

where ν is a uniform kinematic viscosity of the fluid and $\Delta T = T_w - T_\infty$, F , G , H , θ and P are non-dimensional functions in terms of vertical coordinate η . Substituting these transformations into equations (1)–(3) and (9) gives the nonlinear ordinary differential equations,

$$2F + H' = 0, \quad (11)$$

$$F'' - HF' - F^2 + G^2 - MF = 0, \quad (12)$$

$$G'' - HG' - 2FG - MG = 0, \quad (13)$$

$$\frac{\theta''}{Pr} \left[1 + \frac{4}{3R_d} \right] - H\theta' = 0. \quad (14)$$

The transformed boundary conditions are given as

$$\begin{aligned} F = 0, \quad G = 1, \quad H = 0, \quad \theta = 1 \quad \text{at} \quad \eta = 0, \\ F \rightarrow 0, \quad G \rightarrow 0, \quad \theta \rightarrow 0 \quad \text{as} \quad \eta \rightarrow \infty, \end{aligned} \quad (15)$$

where $M = \sigma B_0^2 / (\Omega \rho)$ is the magnetic interaction parameter, $Pr = \nu \rho C_p / \kappa$ is the Prandtl number and $R_d = k^* \kappa / (4\sigma^* T_\infty^3)$ is the radiation parameter. The boundary conditions given by equation (15) imply that the radial (F), the tangential (G) components of velocity and temperature vanish sufficiently far away from the disk, whereas the axial velocity component (H) is anticipated to approach a yet unknown asymptotic limit for sufficiently large η -values.

In the process of integration, the local skin-friction coefficients and the local rate of heat transfer to the surface are calculated. The radial shear stress and tangential shear stress are given by the Newtonian formulae:

$$\begin{aligned} \tau_t &= \left[\mu \left(\frac{\partial v}{\partial z} \right) \right]_{z=0} = \mu G' \Omega (Re)^{\frac{1}{2}}, \\ \tau_r &= \left[\mu \left(\frac{\partial u}{\partial z} \right) + \left(\frac{\partial w}{\partial r} \right) \right]_{z=0} = \mu F' \Omega (Re)^{\frac{1}{2}}, \end{aligned}$$

Hence the tangential and radial skin-frictions are respectively given by

$$(Re)^{\frac{1}{2}} C_{f_t} = G'(0), \quad (16)$$

$$(Re)^{\frac{1}{2}} C_{f_r} = F'(0). \quad (17)$$

Rate of heat transfer is given by

$$\begin{aligned} q &= -\left(\kappa \frac{\partial T}{\partial z}\right) + q_r \\ &= -\left(\kappa \Delta T \left(\frac{\Omega}{\nu}\right)^{\frac{1}{2}} + \frac{16 \sigma^*}{3 k^*} \Delta T (\Delta T \theta + T_\infty)^3 \sqrt{\frac{\Omega}{\nu}}\right) \theta'(0), \end{aligned} \quad (18)$$

Hence the Nusselt number (Nu) is obtained as

$$\frac{(Re)^{-\frac{1}{2}} Nu}{\left(1 + \frac{4}{3R_d} \left[\frac{\theta \Delta T + T_\infty}{T_\infty}\right]^3\right)} = -\theta'(0). \quad (19)$$

where $Re (= \Omega r^2 / \nu)$ is the rotational Reynolds number.

4 Numerical methods for solution

The system of nonlinear differential equations are solved numerically using a standard initial value solver called the Shooting method. The semi infinite integration domain $\eta \in [0, \infty)$ is replaced by a finite domain $\eta \in [0, \eta_\infty)$. In practise, η_∞ should be chosen sufficiently large so that the numerical solution closely approximates the terminal boundary conditions at ∞ . One can easily see that the order of each system of equations exceeds the number of available known boundary conditions. Hence, finding the solution of the system of differential equations is far away from any routine exercise. In practice, we have made initial guesses on $F'(0), G'(0)$ and $\theta'(0)$. Thus, now we have seven boundary condition at $\eta = 0$, which is sufficient to solve the system of equations (11)–(14) as an initial value problem using any standard routine. For this purpose Nachtsheim and Swigert iteration technique has been employed. Thus adopting this numerical technique, a computer program was set up for the solutions of the basic nonlinear differential equations of our problem where the integration technique was adopted as a fourth ordered Runge–Kutta method of integration. Various groups of the parameters M, R_d and Pr were considered in different phases.

5 Results and discussion

Numerical solutions for the problem of hydromagnetic, absorbing, emitting incompressible convective flow of a viscous, electrically conducting gray fluid over a rotating infinite disk in the presence of external uniform magnetic field are obtained. In order to have a

clear insight of the problem, numerical results are obtained by fixing certain values of non-dimensional parameters governing the problem. The parameters are fixed at magnetic parameter (M) 0.0, 0.5, 1.0, and 2.0. Prandtl number (Pr) 0.71, 1, 2.3, and 7.03 (especially for water $Pr = 7.03$) and radiation parameter (R_d) 1, 5, 10, 10^9 .

In order to assess the accuracy of the numerical method, we have compared our radial, tangential and axial velocities and temperature distribution with the previously published work (Ostrach and Thornton, [30, 1958]), for the case of ($Pr = 0.72$, $M = 0.0$, $R_d = 10^9$). The comparison graphs are shown in Fig. 1 and Fig. 2, and a good agreement was observed.

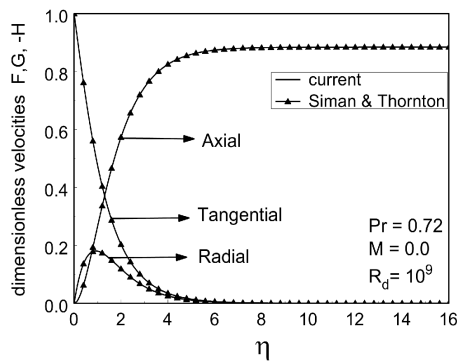


Fig. 2. Comparison of velocity profiles.

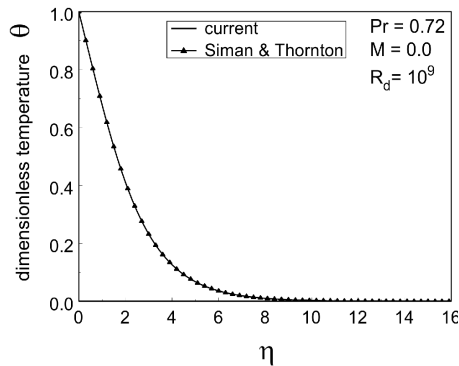


Fig. 3. Comparison of temperature distribution.

The numerical computations adapted in the present investigation, some of our results without magnetic interaction parameter and radiation parameter have been compared with those of Shevchuk [14], in Table 1. The comparison shows excellent agreements, hence an encouragement for the use of the present numerical computations.

Table 1. Comparison of present results with those of Shevchuk [14] with different values of Pr for $M = 0.0$, and $R_d = 1/10^9$.

Pr	Present			Shevchuk [14]		
	$F'(0)$	$-G'(0)$	$-\theta'(0)$	$F'(0)$	$-G'(0)$	$-\theta'(0)$
0.71	0.5102323	0.6159220	0.3259032	-	-	0.3259
0.72	0.5102323	0.6159220	0.3286727	0.5102	0.6159	0.3286
1.0	0.5102323	0.6159220	0.3962484	-	-	0.3963
2.0	0.5102323	0.6159220	0.5652573	-	-	0.5653
7.0	0.5102323	0.6159220	0.9818398	-	-	0.9818

Numerical solutions which are obtained for certain values of physical parameters as mentioned earlier are analysed with the help of graphical illustrations.

From Figs. 4 and 8, it is inferred that the radial velocity, axial velocity respectively gets decelerated due to the effect of magnetic field, this may be due to the fact that the rotating disk acts like a fan drawing axially inwards from the surroundings toward the disk surface. Consequently, the tangential velocity is everywhere lowered and the boundary layer thickness is reduced which is evident from Fig. 5.

The temperature profiles for water ($Pr = 7.03$), showing the effect of variation due to in magnetic parameter is shown through Fig. 7. It is thus apparent that the temperature and the thermal boundary layer thickness increase with M . Imposition of a magnetic field to an electrically conducting fluid creates a drag like force called the Lorentz force. This force has the tendency to slow down the flow around the disk at the expense of increasing its temperature. This is depicted by the decreases in the radial, tangential and axial velocity distribution and increases in the temperature distribution as M increases. However, the effect of M on the temperature distribution θ is not very significant, because the magnetic parameter M does not explicitly occur in the energy equation.

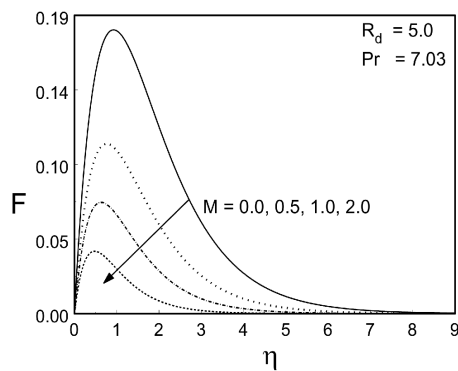


Fig. 4. Effect of magnetic field on radial velocity profile.

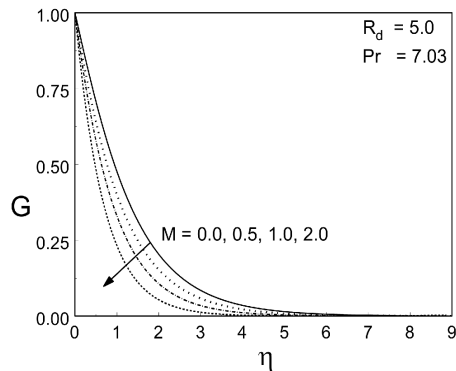


Fig. 5. Effect of magnetic field on tangential velocity profile.

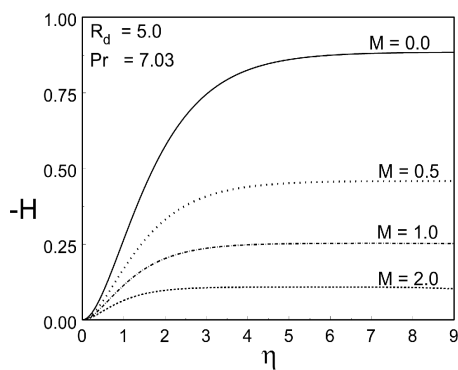


Fig. 6. Effect of magnetic field on axial velocity profile.

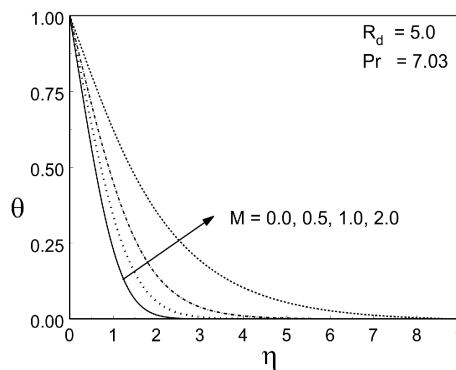


Fig. 7. Effect of magnetic field on temperature distribution.

Fig. 8 portrays the temperature distribution for different values of R_d . From this figure it is noted that the temperature is maximum at the wall and of asymptotically decreases to zero as $\eta \rightarrow \infty$. Further, it is observed from this figure that increase in the radiation parameter decreases the temperature distribution. Also the thermal boundary layer thickness decreases due to increasing R_d which result qualitatively agrees with expectations.

Fig. 9 indicates the effect of Prandtl number Pr on the temperature distribution. It is observed that the temperature decreases with an increase in Prandtl number. This is in agreement with the physical fact that the thermal boundary layer thickness decreases with increasing Pr .

The variation in the surface shear stresses on the radial and tangential directions $F'(0)$, $-G'(0)$ for $0 \leq M \leq 2$ and $R_d = 5$ is shown in Table 2.

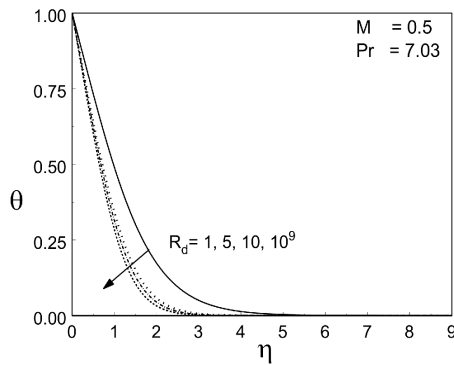


Fig. 8. Effects of R_d on temperature distribution.

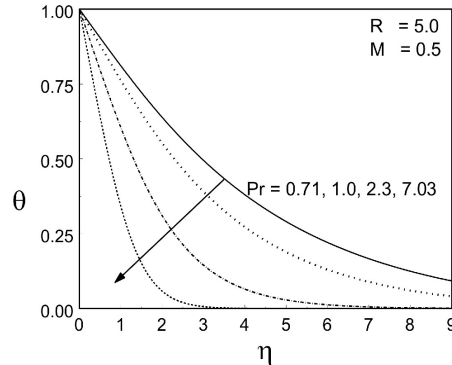


Fig. 9. Effects of Pr on temperature distribution.

Table 2. Variation of $F'(0)$, $-G'(0)$ and $-\theta'(0)$ at the disk surface with M , R_d , and Pr .

M	R_d	Pr	$F'(0)$	$-G'(0)$	$-\theta'(0)$
0.0	5.0	7.03	0.5102326	0.6159220	.8917031
0.5			0.3851327	0.8487239	0.7298949
1.0			0.3092577	1.069053	0.5915729
2.0			0.2305590	1.442094	0.3941377
1.0	1.0	7.03	0.3851327	0.8487239	0.5400746
	5.0		0.3851327	0.8487239	0.7298949
	10.0		0.3851327	0.8487239	0.7686309
	10^9		0.3851327	0.8487239	0.8138981
1.0	5.0	0.71	0.3851327	0.8487239	0.1975103
		1.0	0.3851327	0.8487239	0.2418423
		2.3	0.3851327	0.8487239	0.4091299
		7.03	0.3851327	0.8487239	0.7298949

It is observed from the table that radial and tangential skin friction decrease as M increases. Variations of $-\theta'(0)$ at selected values of M are displayed in Table 2 to exhibit the influence of M on heat transfer. Increasing M tends to decrease the Nusselt number. Also this table indicates the effect of thermal radiation on the heat transfer parameter $-\theta'(0)$. The values of $-\theta'(0)$ increases as the radiation parameter R_d increases.

The variation in the surface shear stresses on the radial and tangential directions $F'(0)$, $-G'(0)$ for $0 \leq M \leq 0.6$ and $R_d = 1$ is shown in Figs. 10 and 11, respectively. For a fixed M , it is seen that they all constant with increasing Pr . It is observed from the figure that radial and tangential skinfriction decrease as M increases. Variations of $-\theta'(0)$ at selected values of M are displayed in Fig. 12 to exhibit the influence of M on the heat transfer. To increase M tends to decrease the Nusselt number. Fig. 13 indicates the effect of thermal radiation on the heat transfer parameter $-\theta'(0)$. The values of $-\theta'(0)$ increases as the radiation parameter R_d increases.

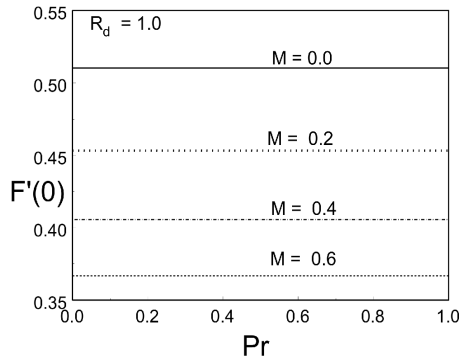


Fig. 10. Radial skin friction coefficient for various M .

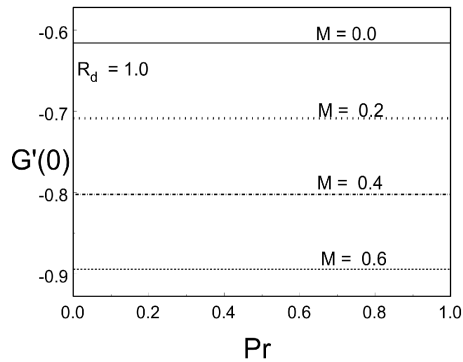


Fig. 11. Tangential skin friction coefficient for various M .

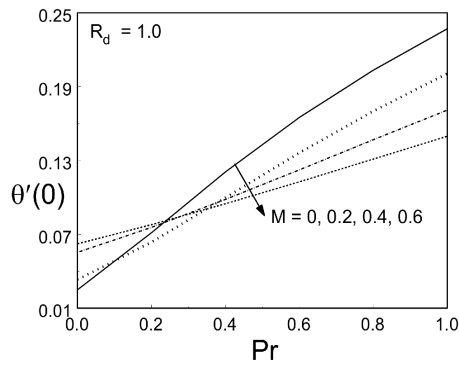


Fig. 12. Influence of M over rate of heat transfer.

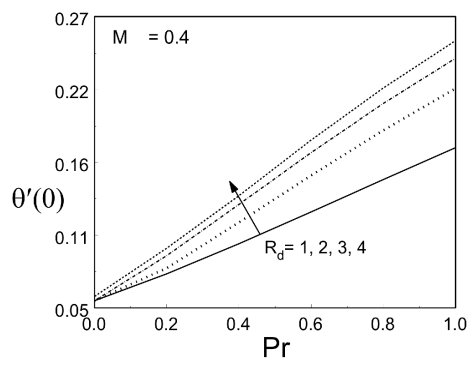


Fig. 13. Influence of R_d over rate of heat transfer.

6 Conclusion

An analysis is performed to study the magnetohydrodynamic forced convection flow of a Newtonian absorbing, emitting incompressible fluid past a rotating infinite disk in the presence of thermal radiation. Using suitable transformations the governing equations are transformed into non-linear ordinary differential equations and are solved numerically by Runge–Kutta based Shooting method. The results are presented for the velocity and temperature profiles, the skin-friction coefficient, and the Nusselt number to reveal the tendency of the solutions. The solutions are found to be governed by three parameters, the magnetic parameter M , Prandtl number Pr and the thermal radiation parameter R_d . Effects of these parameters on the fluid flow and heat transfer characteristics are examined systematically.

The following conclusions are drawn as a result of the computations:

- The effect of the Lorentz force or the usual resistive effect of the magnetic field on the velocity components is apparent. It is also apparent that the magnetic parameter M has increasing effect on temperature distribution.
- It is observed that the temperature distribution decrease for thickness of the increasing values of Pr . In the case of water at 20 °C ($Pr = 7.03$), the thermal boundary layer shows a sharp decrease compared with the effects in electrolyte solution, such as salt water ($Pr = 1.0$) and air ($Pr = 0.71$) at 20 °C.
- As for the effect of the Radiation parameter R_d on the temperature distribution is concerned, it is seen that the temperature distribution decreases with the increasing values of R_d .
- It is observed that the radial skin-friction, the tangential skin friction and the heat transfer coefficient decrease with the increase in the magnetic parameter M .
- The rate of heat transfer increases with the increasing values of R_d .

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