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Prescribed-time practical scaled consensus of multiagent systems via time-based generator approach*

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Abstract. In this paper, we consider the leaderless and leader-following practical scaled consensus problems of multiagent systems (MASs). To achieve leaderless practical scaled consensus, a distributed control protocol is introduced that incorporates with an innovative time-based generator (TBG). Under this protocol, all agents achieve practical scaled consensus within the prescribed-time frame while providing a precise estimation of the practical error. To fulfill practical requirements, we devise two leader-following scaled consensus protocols for both directed detail-balanced graphs and general directed networks. Furthermore, a comprehensive analysis for the convergence of MASs is given by employing the Lyapunov stability theory. Finally, the effectiveness and feasibility of the proposed theoretical results are verified.

Keywords: prescribed-time practical scaled consensus, time-based generator, directed network, multiagent systems.

1 Introduction

With the development of society, more and more complex tasks need to be addressed. The advantages of collaboration among multiple agents in complex tasks have attracted widespread attention from researchers. Cooperative control of MASs has been widely used in many fields, including traffic flow control [26], electrical power grids [21], autonomous underwater vehicles [5], and so on. Consensus is a fundamental issue in cooperative control of MASs in which the main purpose is to design some distributed controllers such that all agents can converge to a common value through local information

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exchange in the network. Compared to single agent systems, MASs have some advantages including increased feasibility and robustness [24].

In recent years, researches were extremely concerned about the traditional consensus of MASs [14, 16, 19, 20], which means that the states of all agents converge to the same value under the control protocol. However, in many practical problems, all agents' states may converge to a prescribed ratio rather than the same value. To solve this problem, a new consensus called the "scaled consensus" was introduced in [13]. Different from traditional consensus, the scaled consensus refers that the ratios of all agents' states converge to some predetermined values. Because scaled consensus can be transformed into traditional consensus, bipartite consensus, and cluster consensus by selecting the appropriate scaled factors, it can be seen as a general form of consensus behaviour. In addition, scaled consensus has received much attention due to its great applications in water distribution systems [11], transcale coordination control of space vehicles [15], etc. In [9], the scaled consensus problem was extended to switching topology, and some sufficient conditions were established to guarantee the exponential convergence of MASs. In [18], the scaled consensus of MASs with communication time-delay was investigated.

The convergence rate is one of the important indexes for controlling system performance. In [1], the finite-time consensus was studied by using nonsmooth gradient flows method. In [3, 17], some continuous state feedbacks and aperiodically intermittent control protocols were proposed to study the finite-time consensus of MASs. In these studies [3, 17], the estimation of the settling time was derived relying on initial conditions of systems, which impedes their practical applications. To solve this issue, some fixed-time consensus protocols were proposed [7, 8] in which the estimation of the settling time is independent of the initial values of the system. However, the settling time cannot be preset arbitrarily because it depends on the control parameters of the system.

With in that mind, the prescribed-time consensus problems were studied by utilizing time-varying function-based controllers in [2, 12]. In [2], by proposing a distributed prescribed-time observer, the prescribed-time consensus was discussed for high-order integrator MASs with both time-invariant and time-varying directed topologies. In [12], the prescribed-time leader-following consensus was investigated for nonlinear MASs in which the nonlinear term satisfies the Lipschitz condition with a time-varying growth rate. In above studies [2, 12], the control gain will be unbounded when the time approaches the predetermined specified instant. To improve this deficiency, the practical prescribed-time consensus was proposed. In [4], the practical prescribed-time consensus was considered for one-order MASs. At present, there are few studies on the prescribed-time practical scaled consensus of MASs.

Motivated by the above discussion, this paper aims to investigate the prescribed-time practical scaled consensus of MASs by using a new time-based generator protocol. Our main contributions are summarized as follow:

(i) A novel TBG with a time-varying gain has been developed, which differs from the ones presented in [4, 10]. Utilizing this newly proposed TBG, a set of innovative prescribed-time practical scaled consensus protocols are designed. In these protocols, the control inputs of the MASs can be effectively regulated by adjusting both the TBG gain and the control parameters, ensuring improved system performance and stability.

(ii) In contrast to fixed-time consensus schemes [7, 8], some prescribed-time consensus protocols are proposed in this paper. By using the proposed control protocols, the settling time for achieving practical scaled consensus is independent of the system parameters and can be preset arbitrarily.

(iii) Unlike traditional consensus approaches [8, 17], this paper investigates practical scaled consensus for MASs by incorporating scaling factors into the control protocols. Both for MASs with and without a leader, sufficient conditions are provided to guarantee the achievement of practical scaled consensus. These conditions can improve the flexibility and applicability of the consensus protocol under different scaling factors.

The remaining sections are organised as follows. First, some preliminaries and the problem statement are given in Section 2. The prescribed-time leaderless and leader-following practical scaled consensus of MASs are studied in Section 3. Some examples are presented to illustrate the effectiveness of the results in Section 4. Finally, conclusions are drawn in Section 5.

Notations. Let $\mathbb{R}$ and $\mathbb{R}^N$ be the one-dimensional real space and the N-dimensional real vector space, respectively. Denote $x = [x_1, x_2, \dots, x_N]^{\mathrm{T}} \in \mathbb{R}^N$ and $\operatorname{sign}(x) = [\operatorname{sign}(x_1), \operatorname{sign}(x_2), \dots, \operatorname{sign}(x_N)]^{\mathrm{T}}$, where sign is the signum function. The p-norm is defined as $\|x\|_p = (|x_1|^p + |x_2|^p + \cdots + |x_N|^p)^{1/p}$, where $p > 0$. $\mathcal{C}^2$ refers to twice continuously differentiable. $\varDelta = \{1, 2, \dots, N\}$. For a vector $q = [q_1, q_2, \dots, q_N]^{\mathrm{T}}$ with $q_i > 0$, $q_{\max} = \max_{i\in\varDelta} q_i$ and $q_{\min} = \min_{i\in\varDelta} q_i$.

2 Preliminaries

2.1 Algebraic graph theory

Consider a MAS consisting of N agents. The communication topology is described by $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$, where $\mathcal{V} = \{v_1, v_2, \dots, v_N\}$ is the vertex set, and $\mathcal{E} = \{(v_i, v_j) \mid v_i, v_j \in \mathcal{V}\}$ is the edge set. Note that direct edge (v_i, v_j) denotes that node v_j can obtain information from v_i, but not necessarily vice versa. The weighted adjacency matrix is $A = [a_{ij}]_{N\times N}$, where $(v_j, v_i) \in \mathcal{E} \Leftrightarrow a_{ij} > 0$, otherwise, $a_{ij} = 0$. Then the Laplacian matrix is $\mathcal{L} = [l_{ij}]_{N\times N}$ with $l_{ij} = -a_{ij}$ for $i \neq j$ and $l_{ii} = \sum_{j=1}^N a_{ij}$. Assuming that there is no self-loop, thus $a_{ii} = 0$. Let $N_i = \{v_j \in \mathcal{V} \mid (v_j, v_i) \in \mathcal{E}\}$ be the neighbor set of node v_i, while $|N_i|$ represents the number of elements in the set N_i. If $\mathcal{G}$ is an undirected graph, then $a_{ij} = a_{ji}$. Specifically, if $\mathcal{G}$ is undirected and connected, the second smallest eigenvalue of $\mathcal{L}$, denoted as $\lambda_2(\mathcal{L})$, is a positive real number. A directed graph $\mathcal{G}$ is said to be detail-balanced if there exists a vector $q = [q_1, q_2, \dots, q_N]^{\mathrm{T}}$ with $q_i > 0$ such that $q_j a_{ji} = q_i a_{ij}$ for $i, j \in \varDelta$. A directed path from node v_i to v_j is a sequence of edges of form $(v_i, v_{i_1}), (v_{i_1}, v_{i_2}), \dots, (v_{i_k}, v_j)$ in the digraph with distinct node $v_{i_l} \in \mathcal{V}$ for $l = 1, 2, \dots, k$. A node is called root if it has a directed path to every other node. A directed graph $\mathcal{G}$ is said to contain a directed spanning tree if it has at least one root.

2.2 The time-based generator

Let $\varphi(t)$ be the TBG function if it has the following properties:

(i) $\varphi(t)$ is at least $\mathcal{C}^2$ on $(0, +\infty)$;
(ii) $\varphi(t)$ is a continuous and nondecreasing function with $\varphi(0) = 0$ and $\varphi(t_f) = 1$ in which $t_f < +\infty$ is a prescribed time instant;
(iii) $\dot{\varphi}(0)$=$\dot{\varphi}(t_f) = 0$, where the derivative of $\varphi(t)$ at $t = 0$ is its right derivative;
(iv) $\varphi(t) = 1$ for $t > t_f$.

Consider a dynamical system, which is described as

$$\dot{g}(t) = h(t), \qquad g(0) = g_0, \tag{1}$$

where $g(t) \in \mathbb{R}$ is the system state. $h(t) \in \mathbb{R}$ is a TBG-based protocol, which is given by

$$h(t) = -\alpha k(t) g(t), \tag{2}$$

where $\alpha > 0$, $k(t)$ is a TBG gain, which is constructed as

$$k(t) = \frac{\dot{\varphi}(t)}{1 - \theta\varphi(t)}, \tag{3}$$

where $\theta \in (0, 1)$ is a positive parameter.

Lemma 1. *For system* (1) *with TBG-based protocols* (2)–(3)*, the solution has the following property*:

$$\lim_{t \to t_f} g(t) = g_0 (1 - \theta)^{\alpha/\theta}.$$

Proof. Consider the following differential equation:

$$\dot{g}(t) = -\alpha k(t) g(t), \qquad g(0) = g_0, \tag{4}$$

where $k(t)$ is designed in (3). The solution for (4) is easily calculated as

$$g(t) = g_0 \bigl(1 - \theta\varphi(t)\bigr)^{\alpha/\theta}.$$

Since $\varphi(t_f) = 1$, we can obtain that $\lim_{t \to t_f} g(t) = g_0(1 - \theta)^{\alpha/\theta}$, which can be reduced to a desire level by choosing an appropriate θ. □

2.3 Problem formulation

In this paper, a MAS consisting of N agents is considered. The dynamics of the ith agent is described by

$$\dot{x}_i(t) = u_i(t), \quad i \in \Delta, \tag{5}$$

where $x_i(t)$ represents the state of the ith agent, and $u_i(t) \in \mathbb{R}^n$ denotes the control input.

Definition 1. (See [26].) The MAS (5) is considered to reach scaled consensus with the scaled factors β_i, where β_i are nonzero constants if for any initial conditions $x_i(0)$,

$$\lim_{t\to\infty} \left\|\beta_i x_i(t) - \beta_j x_j(t)\right\| = 0$$

for $i, j = 1, 2, \dots, N$.

Definition 2. For MAS (5), the prescribed-time leaderless practical scaled consensus is said to be achieved if for any initial states,

$$\lim_{t\to t_f} \left\|\beta_i x_i(t) - \beta_j x_j(t)\right\| \leqslant d;$$
$$\left\|\beta_i x_i(t) - \beta_j x_j(t)\right\| \leqslant d \quad \forall t > t_f;$$
$$\lim_{t\to+\infty} \left\|\beta_i x_i(t) - \beta_j x_j(t)\right\| = 0,$$

where β_i and β_j are scaled factors for $i, j \in \Delta$. $t_f > 0$ is a preset time, which is independent of initial states. d is a positive constant, which represents the desired level.

For a MAS with one leader and N followers agents, the dynamics of leader is described by

$$\dot{x}_0(t) = u_0(t), \tag{6}$$

where $x_0(t) \in \mathbb{R}^n$ and $u_0(t) \in \mathbb{R}^n$ denote the state and the control input of the leader, respectively. The follower's dynamics is given in (5). The communication topology between the leader and N follower agents is represented by the $\bar{\mathcal{G}}$. If the ith agent can able to access the leader's information, then $a_{i0} > 0$; $a_{i0} = 0$ otherwise.

Definition 3. For MASs (5)–(6), the prescribed-time leader-following practical scaled consensus is said to be achieved if for any initial states,

$$\lim_{t\to t_f} \left\|\beta_i x_i(t) - x_0(t)\right\| \leqslant d;$$
$$\left\|\beta_i x_i(t) - x_0(t)\right\| \leqslant d \quad \forall t > t_f;$$
$$\lim_{t\to+\infty} \left\|\beta_i x_i(t) - x_0(t)\right\| = 0,$$

where β_i and β_j are scaled factors for $i, j \in \Delta$. $t_f > 0$ is a preset time, which is independent of initial states. d is a positive constant, which represents the desired level.

Assumption 1. The digraph $\mathcal{G}$ is strongly connected and detail-balanced.

Assumption 2. The leader's input is bounded, i.e., there exists a positive constant $u_{\max}$ such that $\|u_0(t)\| < u_{\max}$.

Assumption 3. The graph $\overline{\mathcal{G}}$ contains a directed spanning tree with leader as the root.

Lemma 2. (*See* [25].) *Suppose that the digraph $\mathcal{G}$ is strongly connected and detail-balanced with positive scalars $q_1, q_2, \dots, q_N$, then the matrix MQ is positive definite in which $Q = \operatorname{diag}(q_1, q_2, \dots, q_N)$, $M = \mathcal{L} + \operatorname{diag}(a_{10}, a_{20}, \dots, a_{N0})$, and $\mathcal{L}$ is the Laplacian matrix of digraph $\mathcal{G}$.*

Lemma 3. (*See* [6].) *If the graph $\overline{\mathcal{G}}$ contains a directed spanning tree, there exists a positive diagonal matrix W such that $U = WM + M^{\mathrm{T}}W > 0$, where $W = \mathrm{diag}(w_1, w_2, \dots, w_N)$ and $w = [w_1, w_2, \dots, w_N]^{\mathrm{T}} = (M^{\mathrm{T}})^{-1}1_N$.*

3 Main results

In this section, three TBG-based protocols are proposed to address the prescribed-time leaderless and leader-following practical scaled consensus issues.

3.1 Prescribed-time leaderless practical scaled consensus

To achieve the prescribed-time leaderless practical scaled consensus, a new TBG-based protocol for the ith agent is proposed as follows:

$$u_i(t) = -\big(\xi_1(t) + 1\big) \sum_{j=1}^{N} \frac{1}{\beta_i} \hat{a}_{ij} \big(\beta_i x_i(t) - \beta_j x_j(t)\big), \tag{7}$$

where $\xi_1(t) = \dot{\varphi}_1(t)/(1 - \theta_1 \varphi_1(t))$, $\varphi_1(t)$ is a TBG function, $\theta_1 \in (0, 1)$. β_i is the nonzero scaled factor. $\hat{a}_{ij} = q_i a_{ij}$, where q_i is the positive detail-balanced scalar for $i \in \Delta$.

Substituting (7) into (5), we can get

$$\dot{x}_i(t) = -\big(\xi_1(t) + 1\big) \sum_{j=1}^{N} \frac{1}{\beta_i} \hat{a}_{ij} \big(\beta_i x_i(t) - \beta_j x_j(t)\big). \tag{8}$$

The compact vector form of (8) is written as

$$\dot{x}(t) = -\big(\xi_1(t) + 1\big)\big(\Lambda^{-1} \hat{\mathcal{L}} \Lambda \otimes I_n\big) x(t),$$

where $\Lambda = \mathrm{diag}(\beta_1, \beta_2, \dots, \beta_N)$, and $\hat{\mathcal{L}}$ is the Laplacian matrix of the graph with adjacency matrix element $\hat{a}_{ij}$.

Remark 1. In the scaled consensus, the scale parameters β_i can be adjusted as needed. If the β_i are the same value, it degenerates to complete consensus. If the β_i are 1 or -1, one can get bipartite consensus. If the values with the same β_i are divided into a group and the scaled parameters are at least two different, the cluster consensus can be achieved. Hence, scaled consensus is a more generalized and comprehensive concept.

Remark 2. In protocol (7), the term of $-\dot{\varphi}_1(t)/(1 - \theta_1 \varphi_1(t)) \sum_{j=1}^{N} (1/\beta_i) \hat{a}_{ij} (\beta_i x_i(t) - \beta_j x_j(t))$ is TBG-based, which is used to ensure that the state disagreement is smaller than a desired level at a prescribed-time. The consensus term of $\sum_{j=1}^{N} (1/\beta_i) \hat{a}_{ij} (\beta_i x_i(t) - \beta_j x_j(t))$ is given to guarantee that the state disagreement converges to zero when $t \to +\infty$. Moreover, the initial control input with protocol (7) is relatively small when the initial state divergence is large. This is due to the TBG attribute of $\dot{\varphi}_1(0) = 0$.

Theorem 1. *If Assumption* 1 *is satisfied, then the MAS* (5) *with protocol* (7) *can achieve prescribed-time leaderless practical scaled consensus. Furthermore, the control input is bounded, and the consensus error satisfies*

$$\lim_{t\to t_{f_1}} \|\beta_i x_i(t) - \beta_j x_j(t)\| \leqslant (N-1)\sqrt{\frac{2N}{\alpha_1}(1-\theta_1)^{\alpha_1/\theta_1} V_1(0)};$$
$$\|\beta_i x_i(t) - \beta_j x_j(t)\| \leqslant (N-1)\sqrt{\frac{2N}{\alpha_1}(1-\theta_1)^{\alpha_1/\theta_1} V_1(0)} \quad \forall t > t_{f_1};$$
$$\lim_{t\to +\infty} \|\beta_i x_i(t) - \beta_j x_j(t)\| = 0,$$

where t_{f_1} *is a designated time instant,* $\alpha_1 = 2\lambda_2(\hat{\mathcal{L}})$, $V_1(0) = x(0)^{\mathrm{T}}(\Lambda\hat{\mathcal{L}}\Lambda \otimes I_n)x(0)/2$.

Proof. Choose the Lyapunov function as

$$V_1(t) = \frac{1}{2}x(t)^{\mathrm{T}}(\Lambda\hat{\mathcal{L}}\Lambda \otimes I_n)x(t).$$

The derivative of $V_1(t)$ is given by

$$\begin{aligned}\dot{V}_1(t) &= -\frac{1}{2}\big(\xi_1(t)+1\big)x(t)^{\mathrm{T}}\big(\Lambda\hat{\mathcal{L}}^{\mathrm{T}}\Lambda^{-1}\Lambda\hat{\mathcal{L}}\Lambda \otimes I_n\big)x(t) \\ &\quad + x(t)^{\mathrm{T}}\big(\Lambda\hat{\mathcal{L}}\Lambda\Lambda^{-1}\hat{\mathcal{L}}\Lambda \otimes I_n\big)x(t) \\ &= -\big(\xi_1(t)+1\big)x(t)^{\mathrm{T}}\big(\Lambda\hat{\mathcal{L}}\hat{\mathcal{L}}\Lambda \otimes I_n\big)x(t) \\ &\leqslant -2\lambda_2(\hat{\mathcal{L}})\xi_1(t)V_1(t) = -\frac{\alpha_1\dot{\varphi}_1}{1-\theta_1\varphi_1}V_1(t).\end{aligned}$$

Using Lemma 1 and the comparison principle, one can get that $V_1(t)$ satisfies $\lim_{t\to t_{f_1}} V_1(t) \leqslant (1-\theta_1)^{\alpha_1/\theta_1}V_1(0)$.

Since

$$\begin{aligned}\frac{1}{2}x(t)^{\mathrm{T}}(\Lambda\hat{\mathcal{L}}\Lambda \otimes I_n)x(t) &\geqslant \frac{1}{2}\lambda_2(\hat{\mathcal{L}})x(t)^{\mathrm{T}}(\Lambda\Lambda \otimes I_n)x(t) \\ &= \frac{1}{2}\lambda_2(\hat{\mathcal{L}})\sum_{i=1}^{N}\beta_i^2\|x_i(t)\|^2,\end{aligned}$$

then it has

$$\frac{1}{2}\sum_{i=1}^{N}\beta_i^2\|x_i(t)\|^2 \leqslant \frac{1}{\lambda_2(\hat{\mathcal{L}})}(1-\theta_1)^{\alpha_1/\theta_1}V_1(0).$$

Furthermore, due to

$$\frac{1}{2}\sum_{i=1}^{N}\beta_i^2\|x_i(t)\|^2 = \frac{1}{2}\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\beta_i^2\|x_i(t)\|^2,$$

then

$$\lim_{t\to t_{f_1}} \|\beta_i x_i(t) - \beta_j x_j(t)\| \leqslant (N-1)\sqrt{\frac{2N}{\alpha_1}(1-\theta_1)^{\alpha_1/\theta_1} V_1(0)}.$$

This means that the state deviation can be reduced to the desired level as long as the appropriate θ_1 is given in protocol (7).

For $t > t_{f_1}$, $\xi_1(t) = 0$, so $\dot{V}_1(t) \leqslant -\alpha_1 V_1(t)$, which implies that $V_1(t)$ is monotonically decreasing. In addition, it has

$$\big\|\beta_i x_i(t) - \beta_j x_j(t)\big\| \leqslant (N-1)\sqrt{\frac{2N}{\alpha_1}(1-\theta_1)^{\alpha_1/\theta_1} V_1(0)} \quad \forall t > t_{f_1}, \tag{9}$$

and $\lim_{t\to+\infty} \|\beta_i x_i(t) - \beta_j x_j(t)\| = 0$. Thus, the prescribed-time leaderless practical scaled consensus problem is solved.

Now, we prove that the control input (7) is bounded. By calculating the derivative of $\xi_1(t)$, one can obtain the following equation:

$$\dot{\xi}_1(t) = \frac{\ddot{\varphi}_1(t)(1-\theta_1\varphi_1(t)) + \theta_1\dot{\varphi}_1(t)}{(1-\theta_1\varphi_1(t))^2}.$$

According to the generalized properties of TBG described in Section 2.2, one can obtain that $\varphi_1(0) = \dot{\varphi}_1(0) = 0$, $\varphi_1(t_{f_1}) = 1$, and $\dot{\varphi}_1(t_{f_1}) = 0$. Furthermore, it can be concluded that $\xi_1(t)$ is bounded for $0 \leqslant t \leqslant t_{f_1}$. Denote $\overline{\xi_1} = \max_{0\leqslant t\leqslant t_{f_1}} \xi_1(t)$.

Due to $\dot{V}_1(t) \leqslant -\alpha_1\dot{\varphi}_1/(1-\theta_1\varphi_1)V_1(t)$, one gets that $V_1(t)$ is nonincreasing, so $V_1(t) \leqslant V_1(0)$, which implies (9). Therefore, the upper bound of u_i in (7), denoted as $\overline{u}_i$, is calculated as

$$\overline{u}_i = \overline{a}|N_i|(\overline{\xi}_1 + 1)\frac{1}{|\beta_{\min}|}(N-1)\sqrt{\frac{2N}{\alpha_1}(1-\theta_1)^{\alpha_1/\theta_1} V_1(0)}, \tag{10}$$

where $\bar{a} = \max_{i,j\in\Delta}\{\hat{a}_{ij}\}$ and $|\beta_{\min}| = \min\{|\beta_1|, |\beta_2|, \dots, |\beta_N|\}$. □

Remark 3. The state disagreement bound

$$(N-1)\sqrt{\frac{2N}{\alpha_1}(1-\theta_1)^{\alpha_1/\theta_1} V_1(0)}$$

depends on the initial states. However, it can be adjusted to the desired level by selecting appropriate parameter θ_1. It is worth noting that the settling time t_{f_1} can be prescribed without dependence on initial conditions. Thus, it can be fascinating and worthy to use the TBG to deal with the practical scaled consensus of MASs.

3.2 Prescribed-time leader-following practical scaled consensus

3.2.1 The MAS with a static leader

In this part, we consider the leader-following MAS (5)–(6) in which the leader is static, i.e., $\dot{x}_0(t) = 0$. Thus, a new TBG-based protocol is proposed as follows:

$$u_i(t) = -\big(\xi_2(t) + 1\big) \times \left(\sum_{j=1}^{N} \frac{1}{\beta_i} a_{ij}\big(\beta_i x_i(t) - \beta_j x_j(t)\big) + \frac{1}{\beta_i} a_{i0}\big(\beta_i x_i(t) - x_0(t)\big)\right), \tag{11}$$

where $\xi_2(t) = \lambda_{\max}(W)\dot{\varphi}_2(t)/(1-\theta_2\varphi_2(t))$, $\varphi_2(t)$ is a TBG function, $\theta_2 \in (0,1)$, and the matrix W is given in Lemma 3.

Let $e_i(t) = \beta_i x_i(t) - x_0(t)$, $i \in \Delta$, and denote $e(t) = [e_1(t), e_2(t), \dots, e_N(t)]^{\mathrm{T}}$. Then the dynamics of the N followers is written in the compact vector form as follows:

$$\dot{e}(t) = -\bigl(\xi_2(t)+1\bigr)(M \otimes I_n)e(t).$$

Theorem 2. *If Assumption* 3 *holds, then the MAS* (5)–(6) *with protocol* (11) *can achieve prescribed-time leader-following practical scaled consensus. Furthermore, the consensus error satisfies*

$$\begin{aligned}
&\lim_{t\to t_{f_2}} \bigl\|\beta_i x_i(t) - x_0(t)\bigr\| \leqslant \sqrt{\frac{2}{\lambda_{\min}(W)}(1-\theta_2)^{\alpha_2/\theta_2} V_2(0)};\\
&\bigl\|\beta_i x_i(t) - x_0(t)\bigr\| \leqslant \sqrt{\frac{2}{\lambda_{\min}(W)}(1-\theta_2)^{\alpha_2/\theta_2} V_2(0)} \quad \forall t > t_{f_2};\\
&\lim_{t\to +\infty} \bigl\|\beta_i x_i(t) - x_0(t)\bigr\| = 0,
\end{aligned} \tag{12}$$

where t_{f_2} is a designated time instant, $\alpha_2 = \lambda_{\min}(U)$, and $V_2(0) = e(0)^{\mathrm{T}}(W \otimes I_n)e(0)/2$.

Proof. Choose the Lyapunov candidate function as

$$V_2(t) = \frac{1}{2}e^{\mathrm{T}}(t)(W \otimes I_n)e(t).$$

The derivative of $V_2(t)$ is given by

$$\begin{aligned}
\dot{V}_2(t) &= e^{\mathrm{T}}(t)(W \otimes I_n)\dot{e}(t)\\
&= -\frac{1}{2}\xi_2(t)e^{\mathrm{T}}(t)\bigl(\bigl(WM + M^{\mathrm{T}}W\bigr) \otimes I_n\bigr)e(t)\\
&\quad - \frac{1}{2}e^{\mathrm{T}}(t)\bigl(\bigl(WM + M^{\mathrm{T}}W\bigr) \otimes I_n\bigr)e(t)\\
&= -\frac{1}{2}\xi_2(t)e^{\mathrm{T}}(t)(U \otimes I_n)e(t) - \frac{1}{2}e^{\mathrm{T}}(t)(U \otimes I_n)e(t)\\
&\leqslant -\frac{\xi_2(t)}{2}\lambda_{\min}(U)e^{\mathrm{T}}(t)e(t) = -\alpha_2\frac{\dot{\varphi}_2(t)}{1-\theta_2\varphi_2(t)}V_2(t),
\end{aligned}$$

where $\alpha_2 = \lambda_{\min}(U)$. Similar to analysis of Theorem 1, $V_2(t)$ satisfies $\lim_{t\to t_{f_2}} V_2(t) \leqslant (1-\theta_2)^{\alpha_2/\theta_2}V_2(0)$. Furthermore, since $V_2(t) = e^{\mathrm{T}}(t)(W\otimes I_n)e(t)/2$ and W is a positive diagonal matrix, then

$$\lambda_{\min}(W)e^{\mathrm{T}}(t)e(t) \leqslant 2(1-\theta_2)^{\alpha_2/\theta_2}V_2(0)$$

and

$$\bigl\|e_i(t)\bigr\|^2 \leqslant \frac{2}{\lambda_{\min}(W)}(1-\theta_2)^{\alpha_2/\theta_2}V_2(0).$$

Thus,

$$\lim_{t\to t_{f_2}}\|e_i(t)\| \leqslant \sqrt{\frac{2}{\lambda_{\min}(W)}(1-\theta_2)^{\alpha_2/\theta_2}V_2(0)}.$$

For $t > t_{f_2}$, $\xi_2(t) = 0$, so $\dot V_2(t) \leqslant -(\alpha_2/\lambda_{\max}(W))V_2(t)$, which implies that $V_2(t)$ is monotonically decreasing. In addition, one can also obtain that

$$\|e_i(t)\| \leqslant \sqrt{\frac{2}{\lambda_{\min}(W)}(1-\theta_2)^{\alpha_2/\theta_2}V_2(0)} \quad \forall t > t_{f_2},$$

and $\lim_{t\to+\infty}\|e_i(t)\| = 0$. Therefore, the consensus error satisfies (12). The proof is completed. □

3.2.2 The MAS with a dynamic leader

When the leader has a dynamic behavior, a new TBG-based protocol is proposed for each follower as follows:

$$\begin{aligned} u_i(t) = &-\big(\xi_3(t)+1\big)\left(\sum_{j=1}^{N}\frac{1}{\beta_i}a_{ij}\big(\beta_i x_i(t)-\beta_j x_j(t)\big)+\frac{1}{\beta_i}a_{i0}\big(\beta_i x_i(t)-x_0(t)\big)\right) \\ &-\mu\frac{1}{\beta_i}\operatorname{sign}\left(\sum_{j=1}^{N}a_{ij}\big(\beta_i x_i(t)-\beta_j x_j(t)\big)+a_{i0}\big(\beta_i x_i(t)-x_0(t)\big)\right), \end{aligned} \tag{13}$$

where $\xi_3(t) = \rho\dot\varphi_3(t)/(1-\theta_3\varphi_3(t))$, $\theta_3 \in (0,1)$, ρ and μ are parameters to be determined later. $\varphi_3(t)$ is a TBG function.

Theorem 3. *Suppose that Assumptions* 1–2 *are satisfied. The MAS* (5)–(6) *with protocol* (13) *can achieve prescribed-time leader-following practical scaled consensus if* $\rho = q_{\max}$ *and* $\mu \geqslant (q_{\max}/q_{\min})u_{\max}$. *Furthermore, the consensus error satisfies*

$$\begin{aligned} &\lim_{t\to t_{f_3}}\|\beta_i x_i(t)-x_0(t)\| \leqslant \sqrt{\frac{2}{\lambda_{\min}(M^{\mathrm T}Q^{-1})}(1-\theta_3)^{\alpha_3/\theta_3}V_3(0)}; \\ &\|\beta_i x_i(t)-x_0(t)\| \leqslant \sqrt{\frac{2}{\lambda_{\min}(M^{\mathrm T}Q^{-1})}(1-\theta_3)^{\alpha_3/\theta_3}V_3(0)} \quad \forall t > t_{f_3}; \\ &\lim_{t\to+\infty}\|\beta_i x_i(t)-x_0(t)\| = 0, \end{aligned} \tag{14}$$

where t_{f_3} *is a designated time instant,* $\alpha_3(t) = 2/(\lambda_{\max}((MQ)^{-1}))$, *the matrices* M *and* Q *are given in Lemma* 2, $V_3(0) = e^{\mathrm T}(0)(M^{\mathrm T}Q^{-1}\otimes I_n)e(0)/2$.

Proof. Let $e_i(t) = \beta_i x_i(t) - x_0(t)$, $i \in \Delta$, and denote $e(t) = [e_1(t), e_2(t), \dots, e_N(t)]^{\mathrm T}$.

Choose the Lyapunov candidate function as

$$V_3(t) = \frac{1}{2}\big((M\otimes I_n)e(t)\big)^{\mathrm T}\big((MQ)^{-1}\otimes I_n\big)\big((M\otimes I_n)e(t)\big).$$

Substituting $u_i(t)$ into (5) and combining with (6), the closed-loop network dynamics is obtained as

$$\begin{aligned}\dot{e}_i(t) = {}& -\beta_i\big(\xi_3(t)+1\big)\left(\sum_{j=1}^{N}\frac{1}{\beta_i}a_{ij}\big(e_i(t)-e_j(t)\big)+\frac{1}{\beta_i}a_{i0}e_i(t)\right)\\ & -\mu\operatorname{sign}\left(\sum_{j=1}^{N}a_{ij}\big(e_i(t)-e_j(t)\big)+a_{i0}e_i(t)\right)-u_0(t).\end{aligned}\tag{15}$$

The above formula is then written in a compact vector form

$$\dot{e}(t) = -\big(\xi_3(t)+1\big)(M\otimes I_n)e(t)-\mu\operatorname{sign}\big((M\otimes I_n)e(t)\big)-\big(1_N\otimes u_0(t)\big).\tag{16}$$

Since the right-hand side of (15) is discontinuous, (16) is understood in a Filippov sense. Then one can get that

$$\begin{aligned}\dot{V}_3(t) &= e^{\mathrm{T}}(t)\big(M^{\mathrm{T}}Q^{-1}\otimes I_n\big)\dot{e}(t)\\ &= e^{\mathrm{T}}(t)\big(M^{\mathrm{T}}Q^{-1}\otimes I_n\big)\big(-\big(\xi_3(t)+1\big)(M\otimes I_n)e(t)\\ &\quad -\mu\operatorname{sign}\big((M\otimes I_n)e(t)\big)-\big(1_N\otimes u_0(t)\big)\big)\\ &\leqslant -\frac{\xi_3(t)}{q_{\max}}\big\|(M\otimes I_n)e(t)\big\|_2^2-(\gamma\mu-\eta)\big\|(M\otimes I_n)e(t)\big\|_1\\ &\leqslant -\frac{\xi_3(t)}{q_{\max}}\big\|(M\otimes I_n)e(t)\big\|_2^2 \leqslant -\frac{2\xi_3(t)}{q_{\max}\lambda_{\max}((MQ)^{-1})}V_3(t)\\ &= -\alpha_3\frac{\dot{\varphi}_3(t)}{1-\theta_3\varphi_3(t)}V_3(t)\end{aligned}$$

for $t\in[0,t_{f_3})$ where $\gamma = 1/q_{\max}$ and $\eta = u_{\max}/q_{\min}$. Using Lemma 1, one can get that $V_3(t)$ satisfies $\lim_{t\to t_{f_3}} V_3(t)\leqslant (1-\theta_3)^{\alpha_3/\theta_3}V_3(0)$.

Furthermore, since $V_3(t) = e^{\mathrm{T}}(t)(M^{\mathrm{T}}Q^{-1}\otimes I_n)e(t)/2$, then

$$e^{\mathrm{T}}(t)\big(M^{\mathrm{T}}Q^{-1}\otimes I_n\big)e(t)\leqslant 2(1-\theta_3)^{\alpha_3/\theta_3}V_3(0)$$

and

$$\big\|e_i(t)\big\|\leqslant\sqrt{\frac{2}{\lambda_{\min}(M^{\mathrm{T}}Q^{-1})}(1-\theta_3)^{\alpha_3/\theta_3}V_3(0)}\quad \forall t>t_{f_3}.$$

One further obtains that

$$\lim_{t\to t_{f_3}}\big\|\beta_i x_i(t)-x_0(t)\big\|\leqslant\sqrt{\frac{2}{\lambda_{\min}(M^{\mathrm{T}}Q^{-1})}(1-\theta_3)^{\alpha_3/\theta_3}V_3(0)}$$

and $\lim_{t\to+\infty}\|e_i(t)\| = 0$.

For $t>t_{f_3}$, $\xi_3(t) = 0$. Similar to analysis of Theorem 1, one can prove that inequality (14) holds. Therefore, the proof is completed. □

Remark 4. It is noteworthy that the static leader is a special case of the dynamic leader. Therefore, protocol (13) and Theorem 3 are also feasible for the case $u_0(t) = 0$.

Remark 5. The positive constant θ in (3) guarantees that $k(t)$ is well-defined when $t \geqslant t_f$. Compared with [10], the form of the $k(t)$ proposed in this paper is $\dot{\varphi}(t)/(1-\theta\varphi(t))$, and the controller $u_i(t)$ does not need to use some properties of the eigenvalue of the Laplacian matrix, which simplifies its design.

Remark 6. In this paper, some new TBG-based protocols are designed to solve prescribed-time practical scaled consensus problems under detail-balanced directed graphs and general directed networks. Moreover, unlike some existing fixed-time protocols, the settling time can be prescribed more accurately by using the new TBG-based protocols. Furthermore, similar to the derivation of formula (10), one can also derive the upper bound of the control input for (11) and (13), respectively.

Remark 7. In [27], to overcome the difficulties caused by the asymmetry property of the Laplacian matrix, the finite-time scaled consensus control scheme was developed by the modified addition of a power integrator method. However, in this paper, we consider the prescribed-time leaderless and leader-following practical scaled consensus problems by designing some new TBG-based protocols in which an explicit bound for the leaderless practical scaled consensus is derived.

Remark 8. In this paper, we propose several distributed protocols based on TBG to address the issues of prescribed-time practical scaled consensus. It is important to note that this work does not consider actuator failures or external disturbances. However, in real-world applications, agents are susceptible to failures, which can lead to a decline in system performance or even unstable. Designing a fault-tolerant control protocol to ensure stable operation of the agents when failures occur is a challenging problem. In [22, 23], the discrete-time and finite-time protocols with fault-tolerant control were proposed to achieve synchronization or consensus of complex networks. These studies provide valuable insights, and we aim to develop some appropriate protocols with fault-tolerant control to solve the scaled consensus problem in future work.

4 Simulation examples

In this section, two examples are given to verify the feasibility and the effectiveness of the proposed leaderless and leader-following practical scaled consensus results. A typical TBG function is set as follows:

$$\varphi(t) = \begin{cases} \frac{10}{4^6}t^6 - \frac{24}{4^5}t^5 + \frac{15}{4^4}t^4, & 0 \leqslant t \leqslant t_f; \\ 1, & t > t_f, \end{cases}$$

where t_f is prescribed-time instant.

Example 1 [*Prescribed-time leaderless practical scaled consensus*]. In this part, consider a MAS with six agents and the dynamics described by (5). The communication topology with connection weights is shown in Fig. 1. The detail-balanced factors $q = [1/24, 1/12, 1/8, 1/6, 1/4, 1/3]^{\mathrm{T}}$.

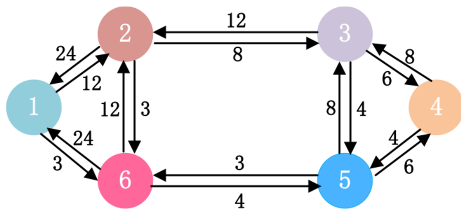
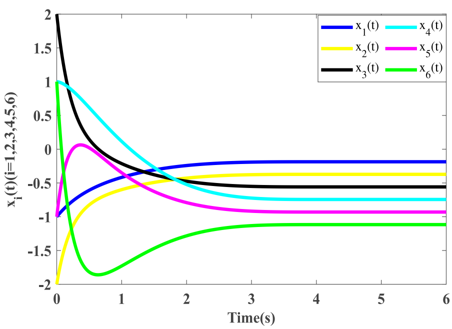
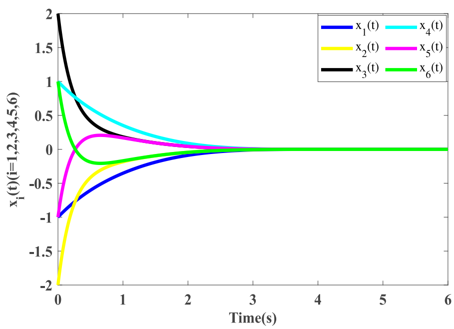


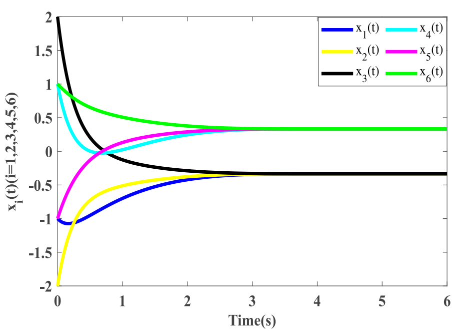
Figure 1. The communication topology of leaderless.



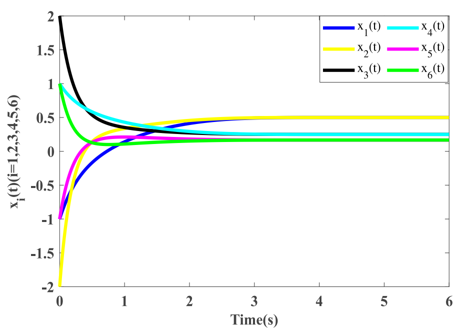
(a) $\beta = (1, 1/2, 1/3, 1/4, 1/5, 1/6)^{\mathrm{T}}$



(b) $\beta = (1, 1, 1, 1, 1, 1)^{\mathrm{T}}$



(c) $\beta = (-1, -1, -1, 1, 1, 1)^{\mathrm{T}}$

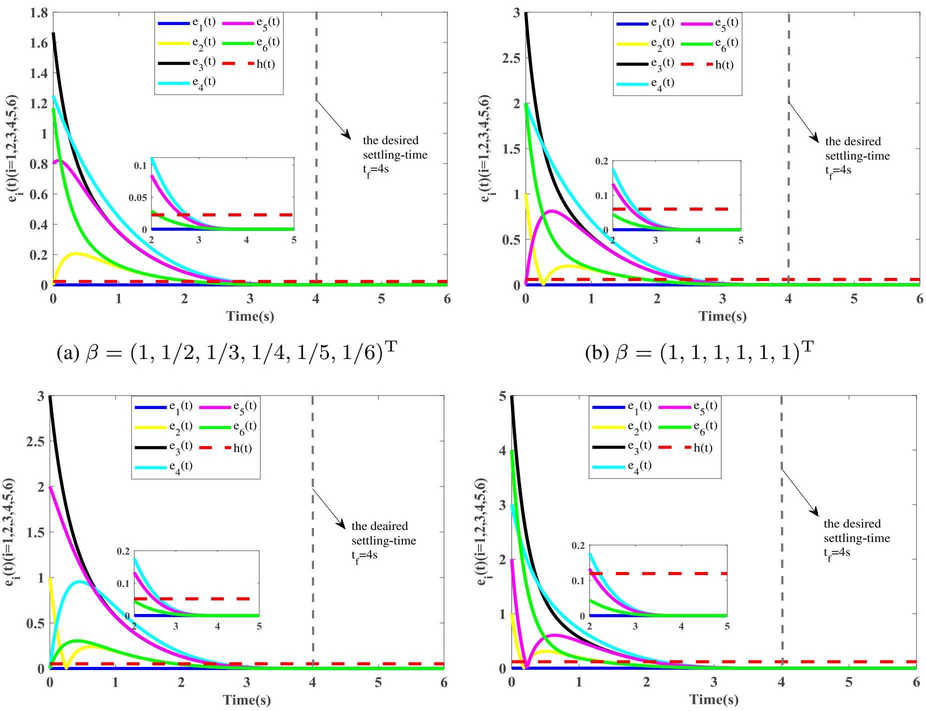


(d) $\beta = (1, 1, 2, 2, 3, 3)^{\mathrm{T}}$

Figure 2. The states with different β values.

Choose the initial conditions as $x(0) = [-1, -2, 2, 1, -1, 1]^{\mathrm{T}}$ and $\theta = 0.99$. Under control protocol (7) with $t_f = 4s$, the simulation results are shown in Figs. 2–3.

Figure 2 shows the state trajectories of system (5), which demonstrates good transient performance. When we choose $\beta = (1, 1, 1, 1, 1, 1)^{\mathrm{T}}$, it can be seen that the state of each of the agents can converge to a common value in Fig. 2(b). However, by choosing different scale ratios, the consensus states of agents are different. When $\beta = (1, 1/2, 1/3, 1/4, 1/5, 1/6)^{\mathrm{T}}$, the state of all the agents did not tend to be of the same value in Fig. 2(a). Furthermore, in order to illustrate the fact that practical scaled consensus is a more general class



(a) $\beta = (1, 1/2, 1/3, 1/4, 1/5, 1/6)^{\mathrm{T}}$ (b) $\beta = (1, 1, 1, 1, 1, 1)^{\mathrm{T}}$

(c) $\beta = (-1, -1, -1, 1, 1, 1)^{\mathrm{T}}$ (d) $\beta = (1, 1, 2, 2, 3, 3)^{\mathrm{T}}$

Figure 3. The errors with different β values.

of consensus, which can include bipartite consensus with $\beta = (-1, -1, -1, 1, 1, 1)^{\mathrm{T}}$ in Fig. 2(c) and cluster consensus $\beta = (1, 1, 2, 2, 3, 3)^{\mathrm{T}}$ in Fig. 2(d).

Moreover, Fig. 3 gives the dynamic evolution of the consensus error under different scaled factors β_i. In Fig. 3(a), the upper bound of the error is $h(t_f) = 0.0024$. When $t \geqslant t_f = 4s$, the error gradually tends to 0. When the selected scaled factors are $\beta = (1, 1, 1, 1, 1, 1)^{\mathrm{T}}$, $\beta = (-1, -1, -1, 1, 1, 1)^{\mathrm{T}}$, and $\beta = (1, 1, 2, 2, 3, 3)^{\mathrm{T}}$ in Figs. 3(b)–3(d), the upper bound of the errors are $h(t_f) = 0.0596$, $h(t_f) = 0.0516$, and $h(t_f) = 0.1192$, respectively. At the same time, when $t \geqslant t_f = 4$ s, the errors gradually tend to 0. The above description illustrates that the leaderless practical scaled consensus can be achieved within the prescribed-time $\mathrm{T}_f = 4$ s, which implies that the protocol (7) is feasible.

Example 2 [*Prescribed-time leader-following practical scaled consensus*]. Consider a MAS with one leader (indexed by 0) and six agents (indexed by $1, \ldots, 6$). The dynamics is described by (5)–(6). The communication topology is shown in Fig. 4.

In the proposed control protocol (13), we choose $\theta = 0.99$, $\mu = 1.6$, $t_f = 1.2s$, and the scaled parameters β are same as the ones in Example 1. The control input of the leader is $u_0(t) = [0.2\sin(5t), 0.2\sin(5t)]^{\mathrm{T}}$. Set the initial states of followers and

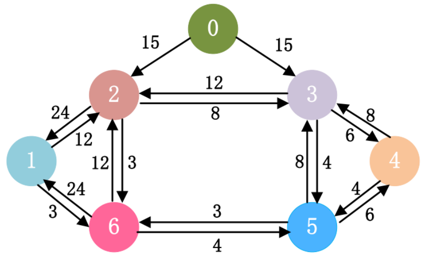
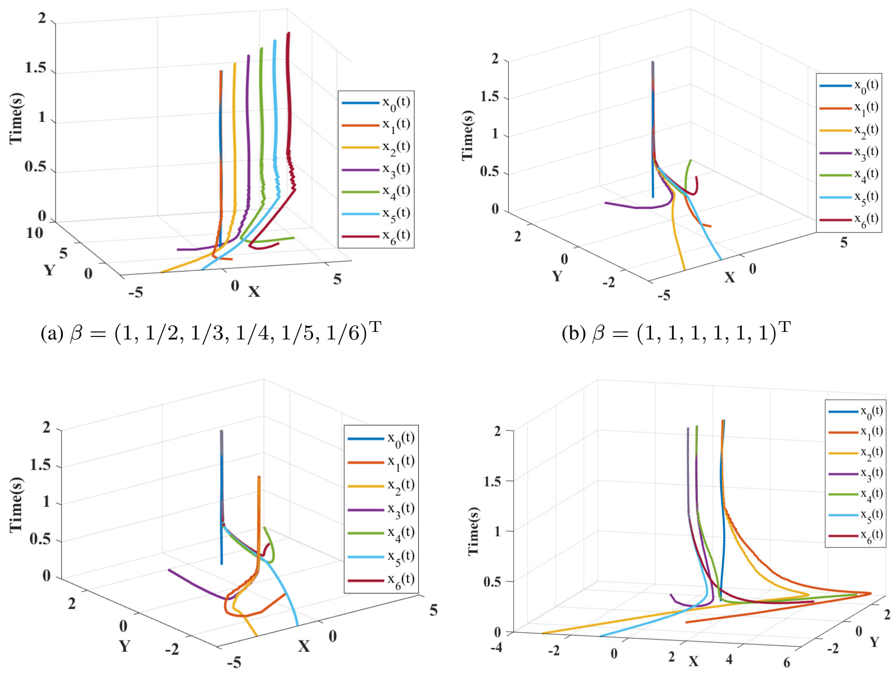


Figure 4. The communication topology of leader-following.



(a) $\beta = (1, 1/2, 1/3, 1/4, 1/5, 1/6)^{\mathrm{T}}$

(b) $\beta = (1, 1, 1, 1, 1, 1)^{\mathrm{T}}$

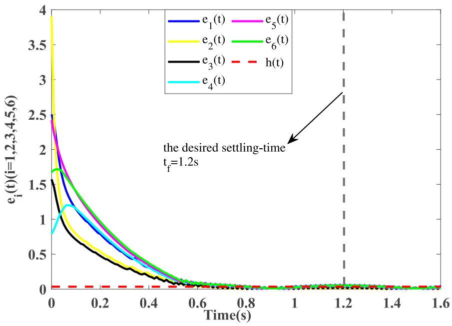
(c) $\beta = (-1, -1, -1, 1, 1, 1)^{\mathrm{T}}$

(d) $\beta = (1, 1, 2, 2, 3, 3)^{\mathrm{T}}$

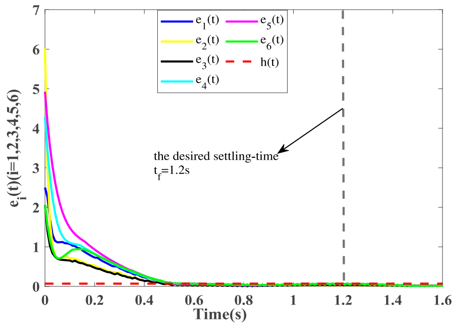
Figure 5. The states with different β values.

leader as $x_1(0) = [-1, 1]^{\mathrm{T}}$, $x_2(0) = [-3, -3]^{\mathrm{T}}$, $x_3(0) = [2, -1]^{\mathrm{T}}$, $x_4(0) = [3, 5]^{\mathrm{T}}$, $x_5(0) = [-3, -1]^{\mathrm{T}}$, $x_6(0) = [2, 4]^{\mathrm{T}}$, and $x_0(0) = [1.5, 1]^{\mathrm{T}}$. The simulation result is shown in Fig. 5.

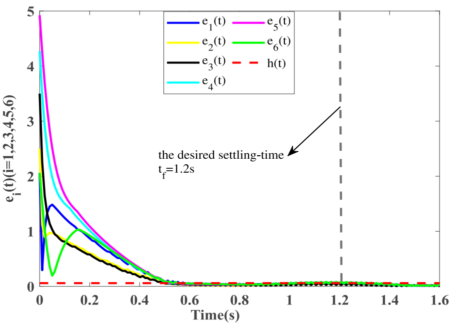
Figure 6 gives the dynamic evolution of the consensus error under different scaled factors β_i. It is found that the upper bounds of the errors are different when the selected scaled factors β_i are different. The tracking errors converge to a neighborhood of zero in the prescribed-time $\mathrm{T}_f = 1.2$ s, which means that the practical scaled consensus is achieved. Consequently, the proposed protocol (13) is effective.



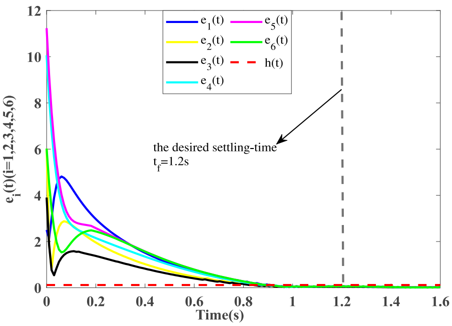
(a) $\beta = (1, 1/2, 1/3, 1/4, 1/5, 1/6)^{\mathrm{T}}$



(b) $\beta = (1, 1, 1, 1, 1, 1)^{\mathrm{T}}$



(c) $\beta = (-1, -1, -1, 1, 1, 1)^{\mathrm{T}}$



(d) $\beta = (1, 1, 2, 2, 3, 3)^{\mathrm{T}}$

Figure 6. The errors with different β values.

5 Conclusion

In this paper, the prescribed-time practical scaled consensus problems were solved for MASs with a new TBG approach in the directed network. Firstly, according to the new TBG method, some new distributed control protocols were proposed. Moreover, we proved that the proposed protocols can ensure all agents achieving practical scaled consensus within the prescribed-time frame while providing a precise estimation of the practical errors. Finally, two examples were used to verify the feasibility of the theory and the effectiveness of the designed control protocols. In our future work, the prescribed-time scaled consensus for MASs with high-order nonlinear dynamics will be considered.

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