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Even in Logic, Laws may Admit of Exceptions

A Survey of some Important Medieval Insights

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Abstract. In this paper it is shown that many medieval logicians recognized that certain 'laws' hold only under certain restrictions. In particular, the basic principles of so-called connexive logic – as they had been put forward by Aristotle, Boethius, and Abelard – hold only for possible, or self-consistent, antecedents, or for non-necessary, or contingent, consequents. A similar restriction applies to the 'law' – possibly put forward by Chrysippus – that each proposition is compatible with itself.
Keywords: Medieval logic, Connexive logic, Aristotle's Theses, Impossible antecedents, Necessary consequents.

Net ir logikoje dėsniai turi išimčių: svarbių įžvalgų Viduramžiais apžvalga

Santrauka. Straipsnyje parodoma, kad daugelis Viduramžių logikų manė, kad kai kurie „dėsniai" galioja tik esant tam tikroms sąlygoms. Pavyzdžiui, Aristotelio, Boecijaus ir Abelardo ginti vadinamosios jungčių logikos (*connexive logic*) baziniai principai galioja tik galimų ar neprieštaringų antecedentų arba nebūtinų ar kontingentiškų konsekventų atžvilgiu. Panašus apribojimas galioja „dėsniui" (kurį galimai gynė Chrisipas), kad kiekvienas teiginys yra suderinamas su pačiu savimi.
Pagrindiniai žodžiai: Viduramžių logika, jungčių logika, Aristotelio tezė, neįmanomi antecedentai, būtini konsekventai.

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1. Introduction

Logical laws are usually considered as being strictly *universal*. For instance, *each* conjunction entails each of its conjuncts:

CONJ 1 $(p \wedge q) \Rightarrow p$ CONJ 2 $(p \wedge q) \Rightarrow q$.

Each disjunction is entailed by each of its disjuncts:

DISJ 1 $p \Rightarrow (p \vee q)$ DISJ 2 $q \Rightarrow (p \vee q)$.

Each proposition entails itself:

INF 1 $(p \Rightarrow p)$[1].

It is hard to imagine how such laws might ever admit of *exceptions*. Yet, medieval logicians developed sophisticated arguments to show that some apparently universal principles hold only under certain restrictions. Some of these arguments turned out to be mere *sophisms*. For instance, the so-called *Burley's paradox* is presented as a refutation of the law of the *transitivity* of the entailment-relation

INF 2 If $(p \Rightarrow q)$ and $(q \Rightarrow r)$, then $(p \Rightarrow r)$,

by arguing:

> For the inference 'I say that you are an ass; therefore, I say that you are an animal' is a good one, and yet something follows from the consequent that does not follow from the antecedent. For it follows: 'I say that you are an animal; therefore, I say the truth'. And yet it does not follow: 'I say that you are an ass; therefore, I say the truth. (Burley 2000: 7)

As it has been shown in Lenzen (2024c), this sophism may rather easily be resolved by noting that the notion of saying is *ambiguous*. If I say *explicitly* that *x* is an ass, then I may perhaps *implicitly* also say that *x* is an animal, and thus I am *implicitly* saying something true. But this does not mean that what I have said *explicitly* would be true!

This paper, however, will not deal with such sophisms but instead focus on certain laws which turned out to be of great importance for modern discussions of *connexive logic*, *paraconsistent logic*, and *relevance logic*. What are these logics, and how are they interrelated? As regards paraconsistent logic, Priest, Tanaka & Weber (2018) explain:

> Contemporary logical orthodoxy has it that, from contradictory premises, anything follows. A logical consequence relation is *explosive* if according to it any arbitrary conclusion *B* is entailed by any arbitrary contradiction $A, \neg A$ (*ex contradictione quodlibet*) […]. Classical logic, and most standard 'non-classical' logics too, […] are explosive. […]
>
> Paraconsistent logic challenges this orthodoxy. A logical relation is said to be *paraconsistent* if it is not explosive. Thus, if a consequence relation is paraconsistent, then even in circum-

[1] As these formulas show, we are using '$\wedge$' and '$\vee$' as symbols for conjunction and disjunction, and '$\Rightarrow$' to symbolize logical implications or entailments. Furthermore, in what follows, '$\neg$', '$\rightarrow$', and '$\leftrightarrow$' symbolize negation, (strict) implication, and (strict) equivalence, respectively, and '□' and '◊' serve as symbols for the modal operators 'necessarily' and 'possibly'. Note that most medieval logicians make no difference between '$p \Rightarrow q$' and '$p \rightarrow q$'.

stances where the available information is inconsistent, the consequence relation does not explode into *triviality*.

Principle 'Ex *contradictione* quodlibet'

ECQ $(p \wedge \neg p) \Rightarrow q$ (for any q)

can be regarded as a special case of the more general principle 'Ex *impossibili* quodlibet'

EIQ If p is impossible, then $(p \Rightarrow q)$ (for any q).

As it will be shown below, many medieval logicians endorsed EIQ together with its 'twin' principle 'Necessarium ad quodlibet':

NAQ If q is necessary, then $(p \Rightarrow q)$ (for any p).

Let it further be noted that, as a corollary of NAQ, any tautology is entailed by any proposition, thus in particular:

TAQ $p \Rightarrow (q \vee \neg q)$ (for any p).

Nowadays, EIQ and NAQ are often referred to as 'paradoxes of strict implication', and they have become the target of criticism not only from proponents of paraconsistent logic but also from proponents of *connexive logic*. According to Wansing (2020), the notion 'connexive logic' was introduced by McCall in order to capture

> […] some ideas about coherence or connection between the premises and the conclusions of valid inferences or between the antecedent and the […] consequent of valid implications. The kind of coherence in question concerns the meaning of implication and negation […]. One basic idea is that no formula provably implies or is implied by its own negation.

The 'passive' version of this idea ('no formula is implied by …') shall here be called Aristotle's Second Thesis, while its 'active' variant ('no formula implies …') is referred to as Abelard's Second Thesis:

ARIST 2 $\neg(\neg q \rightarrow q)$ ABEL 2 $\neg(q \rightarrow \neg q)$.

Similarly, Aristotle's *First* Thesis maintains that no proposition q *is implied* both by proposition p and by p's negation, while its 'active' counterpart says that no proposition p *implies* both q and $\neg q$. The latter principle is usually called '*Boethius*' Thesis', but here it shall be referred to as *Abelard's First* Thesis[2]:

ARIST 1 $\neg((p \rightarrow q) \wedge (\neg p \rightarrow q))$ ABEL 1 $\neg((p \rightarrow q) \wedge (p \rightarrow \neg q))$.

Another characteristic principle of connexive logic goes back to the Stoic logician *Chrysippus* who suggested – as reported in Kneale (1962: 129) – that "[…] a conditional

[2] The terminology 'first thesis' vs. 'second thesis' is justified by the fact that Aristotle first maintained the validity of ARIST 1 and then *derived* this principle from ARIST 2. McCall (2012) speaks of ARIST 1, 2 as '*Aristotle's thesis*' and '*Aristotle's second thesis*' (p. 416), while he refers to ABEL 1, 2 somewhat inconsistently as '*Abelard's* first principle' and as a 'Variant of *Aristotle's* thesis' (p. 417), respectively.

is sound when the contradictory of its consequent is incompatible with its antecedent." Chrysippus furthermore maintained that *each* proposition is compatible with itself. With 'C(p, q)' abbreviating the relation of *compatibility* of two propositions p, q, these two principles can be formalized as follows:

CHRYS 1 $(p \rightarrow q) \Leftrightarrow \neg C(p, \neg q)$ CHRYS 2 $C(p, p)$[3].

In section 2, the various 'quodlibet'-principles will be considered as they were discussed by Peter Abelard and some logic schools of the 12th century. Section 3 focuses on Aristotle's Theses (and their variants) as they were discussed by Robert Kilwardby, Walter Burley, John Buridan, the Pseudo-Scot, and Albert of Saxony. Section 4 deals with Paul of Venice's critique of principle CHRYS 2. The concluding Section 5 is devoted to another issue closely related to the validity of the basic principles of connexive logic, namely, the question whether the truth of an implication $(p \rightarrow q)$ can ever entail (or be entailed by) the truth, or the falsity, of one of the components p, q alone.

2. Ex impossibili Quodlibet & Related Principles

Most medieval logicians endorsed the idea that the modal quality of the antecedent, or premise, of an inference must be *preserved* in the consequent, i.e.:

PRESERV 1 If p entails q and if p is *true*, then q must be true as well;
PRESERV 2 If p entails q and if p is *possible*, then q must be possible, too;
PRESERV 3 If p entails q and if p is *necessary*, then q must be necessary, too.

E.g., Buridan (2015), p. 77, summarized this idea (which basically goes back to Aristotle) by stating that "it is impossible for what is false to follow from truths or what is impossible from the possible or what is not necessary from what is necessary." In view of principle PRESERV 1, the validity of an inference $(p \Rightarrow q)$ *requires* that it is *impossible* that p is true but q is false:

INF 3.1 If $(p \Rightarrow q)$ then $\neg\Diamond(p \wedge \neg q)$.

Moreover, many, though not all, medieval logicians considered the impossibility of $(p \wedge \neg q)$ also as *sufficient* for the validity of an inference, i.e., they endorsed the principle

INF 3.2 If $\neg\Diamond(p \wedge \neg q)$ then $(p \Rightarrow q)$,

and hence, by using INF 3.1, the 'necessary truth preservation account':

INF 3 $(p \Rightarrow q)$ if and only if $\neg(p \wedge \neg q)$.

According to CONJ 1, 2, the conjunction $(p \wedge q)$ entails the single conjuncts; hence, according to PRESERV 2, if $(p \wedge q)$ is *possible*, p and q must be possible, too:

[3] The Chrysippian principles entail the validity of ARIST 2 and ABEL 2; for if, e.g., ARIST 2 would not hold for a certain proposition q, i.e., if $(\neg q \rightarrow q)$, then, according to CHRYS 1, $\neg C(\neg q, \neg q)$ in contradiction to CHRYS 2.

POSS 1 $\Diamond(p \wedge q) \Rightarrow \Diamond p$ POSS 2 $\Diamond(p \wedge q) \Rightarrow \Diamond q$.

In view of the principle of *contraposition* saying that whenever proposition p entails another proposition q, then, the contradictory opposite of q entails the contradictory opposite of p:

CONTRA If $(p \Rightarrow q)$ then $(\neg q \Rightarrow \neg p)$,

it follows from POSS 1 that if p is *impossible*, then, the conjunction $(p \wedge q)$ must be impossible, too:

POSS 3 $\neg\Diamond p \Rightarrow \neg\Diamond(p \wedge q)$ (for any q).

Peter Abelard (1079–1142) was probably the first logician to recognize that POSS 3 in conjunction with INF 3.2 gives rise to the validity of EIQ, i.e., *if* one accepts the impossibility of $(p \wedge \neg q)$ as *sufficient* for $(p \Rightarrow q)$, then any impossible antecedent p entails any consequent q. E.g., the impossible proposition 'Socrates is a stone' entails 'Socrates is an ass', for "it is impossible that Socrates should be a stone, and so impossible that he should be a stone without being an ass"[4]. More generally, whenever p is impossible, then, for any q, it is impossible that p is true *without* q, or, as Abelard put it, "what cannot be true at all, that also cannot be true without the other"[5].

Now, the validity of EIQ (and its counterpart NAQ) is clearly incompatible with the unrestricted version of the connexive thesis ARIST 1, 2, ABEL 1, 2. Therefore Abelard suggested replacing the truth-preservation account, INF 3, by a *stricter* criterion for the validity of an inference:

> There seem to be two necessities of consequences, one in a *larger* sense, if namely that what is maintained in the antecedent cannot be the case without that what is maintained in the consequent; the other in a *narrower* sense, if namely not only the antecedent cannot be true without the consequent, but if also the antecedent requires the consequent *by itself*[6].

As a standard example for a correct implication Abelard mentions "If he is a man, he is an animal." Here, the antecedent requires the consequent 'ex se ipso' since the notion of man *contains* the notion of animal. In contrast, "If he is a man, he is not a stone" is *not* accepted by Abelard as a correct implication, although, of course, it satisfies the weaker criterion INF 3. Abelard argues (1970: 284) that the truth of the latter example rests on our *experience* which shows that the properties 'man' and 'stone' are *disparate*, i.e., they do not simultaneously subsist in one and the same thing. Yet, "the *sense* of the consequent […] is not contained in the *sense* of the antecedent" (Kneale 1962: 218, my emphasis).

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[4] Cf. Abelard (1970: 285). The English translation has been adopted from Kneale (1962: 217).

[5] Cf. Abelard (1970: 285): "quod enim omnino non potest esse [verum] et sine illo non potest esse [verum]."

[6] Cf. Abelard (1970: 284): "[…] altero vero strictior, cum scilicet non solum antecedens absque consequenti non potest esse verum, <sed etiam> ex se ipsum exigit." Kneale (1962: 217) paraphrased this condition as saying that "the antecedent of a true conditional statement requires the consequent *intrinsically*" (my emphasis), while Martin (2004), p. 181, interpreted it as follows: "The antecedent is required to be *relevant* to the consequent in that its truth is genuinely sufficient for that of the consequent and this is guaranteed by the consequent being in some way *contained* in the antecedent" (my emphasis).

In the wake of Abelard, many attempts have been made to elaborate the idea of a 'natural' or a 'relevant' implication, and to develop a full-fledged logic of 'containment'. Until today, however, no real agreement has been reached. Abelard contributed to this enterprise mainly by suggesting that a 'relevant' implication obtains whenever the antecedent refers to a certain *species*, while the consequent refers to the corresponding *kind*. The correctness of such conditionals does not depend on whether the antecedent is true or false. Even *impossible* antecedents can support correct conditionals, e.g., "If Socrates is a pearl, Socrates is a stone," or "If Socrates is an ass, Socrates is an animal"[7].

Somewhen in the 12th century, clever logicians discovered that the principle "Ex *contradictione* quodlibet" can be *proven* by means of the standard laws for disjunction, conjunction, and consequences. In some likelihood, William of Soissons' 'machine', of which John of Salisbury reports in his *Metalogicon*, consisted of such a proof which showed "that from one impossible all impossibles follow"[8]. At any rate, Alexander Neckham's *De Naturis Rerum*, composed around 1180, may be regarded as a *locus classicus* for a full statement of this proof which, after due abstraction, can be formalized as follows[9]:

1. $(p \wedge \neg p) \Rightarrow p$ — Conj 1
2. $(p \wedge \neg p) \Rightarrow \neg p$ — Conj 2
3. $p \Rightarrow (p \vee q)$ — Disj 1
4. $(p \wedge \neg p) \Rightarrow (p \vee q)$ — Inf 2(1,3)
5. $(p \vee q), \neg p \Rightarrow q$ — Disjunctive Syllogism
6. $(p \wedge \neg p) \Rightarrow q$ — Inf 2(2,4,5).

The decisive 'new' element in this proof consists of the so-called *disjunctive syllogism*

Disj 3 $\quad (p \vee q), \neg p \Rightarrow q \quad$ Disj 4 $\quad (p \vee q), \neg q \Rightarrow p$,

which was formulated, e.g., in the *Avranches text* as follows: "from a disjunction and the destruction of one of its parts the positing of the remaining part follows"[10].

The author of the *Avranches text* attempted to prove even the *stronger* principle 'Ex *impossibili* quodlibet'. Starting from the 'impossible' assumption "Socrates is an ass," he first tried to derive an explicit contradiction of the form "Socrates is a man" and "Socrates is not a man" before he continued with an argument which basically has the same structure as the above-presented proof-sequence 1.–6. The derivation of the contradiction proceeds as follows:

[7] Cf. Abelard (1927: 329), and Abelard (1970: 346).

[8] Cf. Kneale (1962: 201), and Martin (1986: 565).

[9] Cf. Neckham (1863: 288–289).

[10] Cf. Iwakuma (1993: 136): "ex disiuncta enim et destructa parte illius sequitur positio reliquae partis." Later medieval authors provided more elegant formulations, such as "from a disjunction together with the contradictory of one part the remaining part follows" (Burley 2000: 207). Buridan (1976: 37), justifies the corresponding inference by saying: "Et iste syllogismus tenet per locum a diuisione, quia duobus positis sub disiunctione si alterum interimatur reliquum concludetur." Read translated this as follows: "This is a case of disjunctive syllogism – that from a disjunction, if either [disjunct] is denied, the other may be inferred" (Buridan 2015: 79).

(i) If Socrates is an ass, Socrates is
(ii) If Socrates is, Socrates is Socrates
(iii) If Socrates is Socrates, Socrates is a man
(iv) If Socrates is an ass, Socrates is not a man.

Some steps of this argument appear somewhat dubious, however. In particular, it is hard to accept the logical validity of (iii) since the antecedent "Socrates is Socrates" is just an empty tautology whereas the consequent "Socrates is a man" appears to be a *contingent* truth. At any rate, the entire argument is not *formally* valid; it fails, e.g., if the name 'Socrates' is replaced by the medieval dummy name of an ass, 'Brunellus'[11].

Similarly, John Buridan (ca. 1300–1358) thought that, besides "Ex *contradictione* quodlibet," the stronger principle "Ex *impossibili* quodlibet" might be provable. He noticed that EIQ can be formally derived from ECQ by means of the following principle of the "redundancy of necessities":

BURI If $p_1, p_2 \Rightarrow q$, and if p_1 is necessary, then $p_2 \Rightarrow q$.

Buridan argued that:

> […] every consequence from an impossible antecedent may be reduced to a *formal* consequence by *the addition of some necessity*. For if the antecedent is impossible, its contradictory is necessary, and by adding it we obtain anything by a formal consequence, as was said. (Buridan 2015: 80; my emphasis)

Clearly, if p is impossible, then $\neg p$ is necessary, and, according to ECQ, from p and $\neg p$ one can *formally* infer any q. Thus, if principle BURI would be valid, one may drop the premise $\neg p$ and conclude that q follows from p alone. However, as was shown in Lenzen (2024a), Buridan's 'proof' of BURI turned out to be somewhat *circular* insofar as it *presupposed* the validity of EIQ.

3. Aristotle's and Abelard's Theses

Before analysing the arguments that medieval logicians put forward in favour of, or against, the connexive theses, a word on terminology may be in order. As it was argued in Lenzen (2022), a proper understanding of the *history* of connexive logic requires that one distinguishes two forms of connexivism, viz., *restricted* or 'humble' connexivism on the one hand, and *unrestricted* or 'hardcore' connexivism on the other. The term 'humble connexivism' was coined in Kapsner (2019), while 'hardcore connexivism' was introduced in Lenzen (2019). Wansing & Omori (2024: 4) criticized this terminology:

> Note that Wolfgang Lenzen […] proposes another language reform, namely to call defenders of connexive logic 'hardcore connexivists'. We advise not to follow this proposal. The suggestion seems to be motivated by an opinion about what Lenzen considers to be non-hardcore conceptions of 'normal' conditionals.

[11] A similar remark also applies to the diverse 'proofs' of EIQ in some 12th century tracts discussed in Binini (2024). Interestingly, the 'proof' in the *Tractatus Vaticanus* is almost identical with (i)–(iv).

Furthermore, Kapsner's notion 'humble connexivity' is criticized because Wansing and Omori consider the expression 'humble' as "morally tinged […] (suggesting that unrestrictedly connexive logics are immodest)" (ibid.: 17). Let it be emphasized once again that the term 'hardcore' was never meant to have an offending connotation; it was just introduced to characterize the position that connexive principles like Arist 1, 2 or Abel 1, 2 hold *without any restriction*, while 'humble' connexivity is ready to admit that *there are some exceptions*, namely, impossible antecedents and/or necessary consequents. As this section will confirm, a great part of the medieval discussions of Aristotle's Theses, Boethius' Thesis, and Abelard's Theses is concerned with exactly this issue: 'humble' or 'hardcore'.

In Section 1 above, it was claimed that Ecq, Eiq, Naq, and Taq stand in conflict with the basic ideas of paraconsistent logic, relevance logic, *and connexive logic*. The third conflict can now be described more precisely as follows: The 'quodlibet' principles are incompatible with '*hardcore*' connexivism; they *define* the *exceptions* for which, according to '*humble*' connexivism, Aristotle's Theses and Abelard's Theses fail to hold, namely, impossible (in particular, self-contradictory) antecedents, and necessary (in particular, tautological) consequents.

3.1. Some attempts to defend 'hardcore' connexivism

The controversy between Abelard and some logic schools of the 12th century was first described in Martin's pioneering papers (1986), (1987). As it was also summarized and evaluated in Lenzen (2023), (2024b), a brief survey may suffice here. In the *Introductiones Montane Minores*, the following argument which is usually attributed to Alberic of Paris was presented:

> It can be proven that a proposition infers its own contradictory in this way: if Socrates is a man and not an animal, Socrates is not an animal; and if Socrates is not an animal, Socrates is not a man; if Socrates is not a man, Socrates is not a man and not an animal[12].

With '$M(x)$' and '$A(x)$' symbolizing the predicates of being a *man* and being an *animal*, respectively, and with 's' abbreviating the name 'Socrates', this proof can be formalized as follows:

1.	$M(s) \wedge \neg A(s) \Rightarrow \neg A(\mathrm{s})$	Conj 2
2.	$M(s) \Rightarrow A(s)$	Analytically true
3.	$\neg A(s) \Rightarrow \neg M(s)$	Contra(2)
4.	$M(s) \wedge \neg A(s) \Rightarrow M(s)$	Conj 1
5.	$\neg M(s) \Rightarrow \neg(M(s) \wedge \neg A(s))$	Contra(4)
6.	$M(s) \wedge \neg A(s) \Rightarrow \neg(M(s) \wedge \neg A(s))$	Inf 2(1,3,5).

[12] Cf. De Rijk 1967, II/2, p. 65–66: "Sicque probari potest quod una propositio suam inferat contradictoriam hoc modo: si Socrates est homo et non est animal, Socrates non est animal; et si Socrates non est animal, Socrates non est homo; si Socrates non est homo, non est Socrates homo et non animal."

Since Abelard endorsed all logical principles used in this derivation, "[...] confronted with this argument Master Peter essentially threw up his hands and granted its necessity" (Martin 1987: 395). However, other logic schools of that time attempted to 'save' the connexive principles by casting into doubt the law of the transitivity of the inference relation, Inf 2, or the laws of conjunction, Conj 1, 2[13].

The adherents of the school of the *Montanae* rejected principle Inf 2 because it allows to derive the 'inconveniency' that if no proposition exists, then no human being exists, either, for one could argue:

> […] if there exist human beings, the proposition which says 'There exist human beings' is true. And if this proposition is true, then some proposition, namely the proposition which says so, exists. Hence, if human beings exist, some proposition exists according to the above-mentioned rule [of transitivity]. But further, if no proposition exists, then, according to the principle of destroying the consequent [i.e., the law of contraposition], no human being exists either[14].

This argument is far from convincing, however. First, it is not at all clear whether the conclusion "If no proposition exists, no human being exists" is a genuine 'inconveniency'. After all, it might be objected that propositions are *products* of the human mind so that, if these products have vanished, it must be due to the fact that the producers have vanished, too. Second, even if the conclusion is considered as a kind of contradiction, it is not at all clear which assumption should be made responsible for its derivability. E.g., even the very first step of the argument of the *Montanae* might be doubted. Thus, Buridan considered the inference "No human being exists;" therefore, "The proposition that no human being exists is *true*," as *unwarranted*[15]. Anyway, it remains unclear why the *transitivity* of the inference-relation should be the real culprit, and, moreover, 'hardcore' connexivism might be refuted even without this law. Thus, if, in the above proof, predicate '$A(x)$' is replaced by '$M(x)$', Conj 1 and Conj 2 allow to prove that the self-contradictory antecedent $M(s) \wedge \neg M(s)$ entails both $M(s)$ and $\neg M(s)$, in contradiction to Abel 2.

Martin remarked in (1986: 570), that "there is some evidence that Abelard's followers, the *Nominales*, held that only the positive conjunct is entailed by such a conjunction." In (1987: 397), Martin explained "that the *Nominales* adopted the view that from an affirmation and a negation only the affirmative conjunct follows." That means that Step 1 of the above proof is *rejected* because the conjunction of an affirmative proposition *p*

[13] Apparently, no medieval logician ever doubted the validity of the law of contraposition which in the *Introductiones Montane Minores* is formulated as the rule: "si aliquid infert aliud, destructo consequenti destruitur antecedens" (De Rijk 1967: 64).

[14] Cf. De Rijk (1967: 65): "[…] si sic esset hec regula, aperte posset probari secundum hoc tale inconveniens quod si nulla propositio esset, nullus humo esset. Dicit enim Aristotelis in *Cathegoriis* quod si homo est, vera est propositio qua dicitur '*homo est*'. Et si vera est propositio '*homo est*', propositio qua id dicitur, aliqua est; quare si homo est, propositio aliqua est per regulam suprapositam: […] Et si quia homo est, aliqua propositio est, tunc si nulla propositio est, nec homo est. A destructo consequenti."

[15] According to Buridan, the principle "that every proposition entails its own truth, is not strictly speaking correct. From 'A man exists' it does not follow that 'A man exists' is true; for a man could exist even though there were no propositions […]" (Hughes 1982: 38).

and a negative proposition q is supposed to entail only p but not q! This strange idea was described in Martin (2004: 198–9), at some greater length:

> They said that 'if Socrates is human and Socrates is not an animal, then Socrates is not an animal' does not hold because a negation is not so powerful (*vehemens*) when joined with an affirmation as it is when it is alone, and something follows from a negation alone which does not follow from it when it is conjoined with an affirmation.

This view is meant to be supported by the example "from the negation '*Socrates doesn't dispute*' when conjoined with the affirmation '*when Plato is reading*' it doesn't follow '*Socrates doesn't discuss with anybody*', although this follows when the proposition is put forward *per se*" (ibid.). Now, this example, of course, is entirely *correct*. If the proposition "Socrates doesn't dispute" is *restricted* by adding a qualification like "*when* Plato is reading," it no longer retains the inferential power of its unrestricted counterpart. But in Alberic's proof (1.–6.), the situation is very different. The second, negative conjunct of "Socrates is a man *and* Socrates is not an animal" does *not restrict* the first, affirmative conjunct. The contents of both conjuncts are simply conjoined but not diminished or relativized. As it was explained in Lenzen (2023), this point had basically been acknowledged also by the author of the *Introductiones Montane minores* himself.

A more solid argument against the conclusiveness of Alberic's argument was put forward by the *Porretani*, i.e., the followers of Gilbert of Poitiers (1076–1154). They argued that the general laws of conjunctions were "[…] at fault, since what we assert with such conditionals is that the conjuncts are *conjointly* sufficient for the consequent but neither alone is sufficient" (Martin 1986: 570/1, my emphasis). Their reason for rejecting Conj 1, 2 was put forward in the *Compendium Logicae*:

> This is because it is a general principle both with regard to consecution and to inference [that it follows] only if the cause of the consequent or the conclusion is preposed to the consequent or conclusion. What indeed is asserted in 'If Socrates is a man and the Seine flows through Paris, then Socrates is an animal'? And what relevance does one of a pair of coupled antecedents have to the consequent when only the other is the cause? Thus in 'If Socrates is a man and an ass, then Socrates is a man', isn't 'Socrates is an ass', which is not a cause, preposed as a cause[16]?

The Porretani thus advocate a causal interpretation of conditionals according to which $(p \to q)$ is true if and only if the antecedent is the *cause* of the consequent. The notion of cause, however, is somewhat unorthodox in so far as proposition p (or the state of affairs described by p) is apparently accepted as the 'cause' of itself. As the second example indicates, Socrates's being a man 'causes', and hence implies, that Socrates is a man. Thus, Porretanian implication may be assumed to be reflexive (and probably also transitive). Yet, it fails to satisfy the laws Conj 1, 2, because the proposition "The Seine flows through Paris" is totally *irrelevant* for inferring "Socrates is an animal" from "Socrates is a man $\wedge$ The Seine flows through Paris." Similarly, the counterfactual assumption "Socrates is

[16] This rather free translation of the Latin text in Ebbesen, Fredborg and Nielsen (1983: 22), has been adopted from Martin (1987: 397).

an ass" is totally irrelevant for deriving "Socrates is an animal" from 'Socrates is a man Socrates is an ass'. In the opinion of the Porretani, whoever holds that if ($p \to q$), then also ($p \wedge r \to q$), commits, as they say, the fallacy '*non causa ut causa*': The conjunctive antecedent ($p \wedge r$) is 'preposed' as the *cause* of the consequent q, but the true cause of q is p alone, while ($p \wedge r$) is a '*not-cause*'. As Martin (1987: 397/8) remarked, the reservations of the Porretani concern "exactly the point made by Everett Nelson in his account for the intensional relationship holding between the antecedent and consequent of a true conditional." For reasons of space, a closer discussion of this point cannot be given here. Let it only be pointed out that Nelson's 'relevantist' rejection of the laws of conjunction, as put forward in Nelson (1930), was critically examined in Lenzen (2024d).

3.2. Arguments in favour of 'humble' connexivism

In his extensive commentary on Aristotle's *Prior Analytics*, Robert Kilwardby (1222–1277) tried to defend Aristotle's connexive theses at all costs, although he was well aware of the fact that they stand in conflict with principles Eiq and Naq. On the one hand, one easily gets counter-examples against Arist 1:

> For it seems that one and the same thing does follow from the same thing's being so and not being so, because if you are sitting then God exists and if you are not sitting then God exists, because the necessary follows from anything. (Thom & Scott 2015: 1141)

On the other hand, Arist 2 does not appear to hold without restriction, either:

> […] since one opposite may well follow from another, as in 'If you are an ass, you are not an ass', because anything follows from the impossible and the necessary follows from anything. (ibid.: 1143)

To this Kilwardby objected that one has to distinguish two types of inferences: *essential* or *natural* inferences, vs. merely *incidental* or *accidental* inferences. Inferences based on Eiq and Naq are only incidentally valid, while Aristotle's theses have to be interpreted as holding for natural inferences.

However, Kilwardby clearly saw that counter-examples to Arist 1, 2 need not resort to principles Eiq and Naq, but may also be achieved, e.g., by application of the usual laws of disjunction. Thus,

> […] a disjunctive follows from either of its parts, and in a natural inference. Hence it follows 'If you are sitting, you are sitting or you are not sitting', and 'If you are not sitting, you are sitting or you are not sitting'. And thus one and the same thing follows in a natural inference, and thus of necessity, from the same thing's being so and not being so. (ibid.: 1141)

More generally, the laws Disj 1, 2 entail the following special instances:

Disj 5 $\quad p \Rightarrow (p \vee \neg p) \quad$ Disj 6 $\quad \neg p \Rightarrow (p \vee \neg p)$,

which shows that one and the same proposition can be 'naturally' inferred from opposites. However, Kilwardby was not yet willing to give in but instead suggested to take another distinction into account:

> […] the same thing can follow in two ways, viz. either by virtue of the same thing in it […] or by virtue of different things in it […]. So Aristotle understands that something does not follow of necessity from the same thing's being so and not being so, in a natural inference, *and by virtue of the same thing* (ibid.: 1141–1143, my emphasis).

The latter condition apparently was introduced entirely *ad hoc*[17]. This had to be granted even by McCall who, otherwise, would have loved to claim Kilwardby as a proponent of 'hardcore' connexivism:

> Kilwardby tries to defend Aristotle's position nevertheless, saying that the Philosopher intended only to deny that the same proposition could follow from two contradictories "in virtue of the same part of itself" (*gratia eiusdem in ipso*). But it is doubtful that Aristotle intended any such thing, and Kilwardby seems to be leaning over backwards here. It appears we must accept the fact that the type of implication for which Aristotle's thesis holds cannot consistently admit of conditionals of the form "if *p*, then either *p* or *q*". (McCall 2012: 418)

McCall failed to mention that, besides the laws of *disjunction*, also the laws of *conjunction* seriously threaten the validity of Aristotle's Theses. Clearly, Conj 1, 2 immediately entail the special instances

Conj 3 $(p \wedge \neg p) \Rightarrow p$ Conj 4 $(p \wedge \neg p) \Rightarrow \neg p$.

Hence, two contradictories do follow from one and the same proposition, and these entailments cannot be blamed for not satisfying the *ad hoc* condition 'in virtue of the same'. As a corollary of Conj 3 and Disj 5, one further obtains

Kilw 1 $(p \wedge \neg p) \Rightarrow (p \vee \neg p)$.

In view of the 'De Morgan' laws (plus the law of double negation), this principle can be transformed into

Kilw 2 $(p \wedge \neg p) \Rightarrow \neg(p \wedge \neg p)$.

Thus, in the end also Kilwardby had to admit: "So it should be granted that from the impossible its opposite follows, and the necessary follows from its opposite" (Thom & Scott 2015: 1145).

The next author to be considered in this section is Walter Burley (ca. 1275–1345). In the 'Longer Treatise' of *On the Purity of the Art of Logic*, he noted that "every conditional is true in which an antecedent that includes opposites implies its contradictory," and he proved this principle as follows:

> (313) If some proposition includes opposites, it implies either of them. Since therefore, from the opposite of a consequent there follows the opposite of its antecedent, from the opposite of either of those contradictory consequents there must follow the contradictory of the antecedent. Since therefore, the opposite of either one follows from the same antecedent, and whatever follows from the consequent follows from the antecedent, from that antecedent there must follow its contradictory. (Burley 2000: 157)

[17] Nevertheless, Johnston (2019) tried to develop a system of 'hardcore' connexive logic based on exactly this *ad hoc* proposal; for a critical comment, cf. Lenzen (2020).

Now, according to CONJ 3, 4, there are propositions which entail opposites. Hence, in view of principle IMPL 6, there are also propositions which entail their own negation. Thus, Burley stated the following diagnosis:

> Suppose someone says contradictories do not follow from the same antecedent. For in that case the same thing would follow from contradictories which seems to be contrary to the Philosopher in *Prior Analytics* […]. I say that the same consequent does not follow from the same antecedent affirmed and denied, *unless the opposite of that consequent includes contradictories* [i.e., unless the consequent is necessary]. And this is how Aristotle's statement has to be understood. (Burley 2000: 160, my emphasis)

Similarly, in his *In Librum Secundum Priorum Analyticorum Aristotelis Quaestiones*, Pseudo-Scot extensively discussed the question "Whether one and the same proposition can follow from both of two contradictory propositions." In order to arrive at a complete answer, the author distinguishes three types of inferences, those which are *formally* valid, *simply* valid, or only valid *as of now* ('ut nunc'). The inference ($p \Rightarrow q$) is *simply* valid if it is impossible that p is true while q is false. This amounts to our earlier criterium INF 3. As a typical example, Pseudo-Scotus mentions the inference "Some human is running, therefore some animal is running." The validity of this inference depends on the meaning of the terms 'human' and 'animal'. In contrast, the inference ($p \Rightarrow q$) is *formally* valid if its validity does not depend on the specific terms occurring in p and q, so that it remains valid if the terms are replaced by arbitrary other terms. As Pseudo-Scot explains, each simply valid inference can be transformed into a formally valid inference by the addition of an appropriate premise which must be *necessarily* true. Thus, in the above-presented example, when "Every human is an animal" is added to "Some human is running," the conclusion "Some animal is running" follows formally[18].

Having made these distinctions, Pseudo-Scotus draws various conclusions. First, as stated in principle ECQ, any *self-contradictory* proposition like, e.g., a conjunction of type ($p \wedge \neg p$) *formally* entails any (other) proposition: "Thus, it follows 'Socrates is running and Socrates is not running, therefore you are in Rome'". Second, any *impossible* proposition entails any (other) proposition, but these inferences are only *simply* and not formally valid[19]. Accordingly, in view of the law of *contraposition*, every *necessary* proposition follows *simply*, but not formally, from any (other) proposition[20]. As a corollary, any necessary proposition p is *simply* entailed by both of the two contradictory propositions q and $\neg q$. Interestingly, Pseudo-Scot did not prove this corollary in its full generality but

[18] Similarly, ($p \Rightarrow q$) is valid *as of now* if it can be transformed into a formally valid inference by the addition of a premise which, however, need not be *necessarily* true but only "true as of now," i.e., true as a matter of fact: "[…] dicitur consequentia sequi *ut nunc solum*, quando medium per quod consequentia fit evidens non est necessarium, sed contingens" (Duns Scotus 1891: 184a).

[19] Cf. Duns Scotus (1891: 184b): "Secunda conclusio, ad quodlibet impossibile, sequitur quaelibet alia propositio, non formaliter, sed simpliciter tantum."

[20] Cf. Duns Scotus (1891: 184b): "Ex istis sequitur illa regula, quod *Necessarium sequitur ad quodlibet*." The law of contraposition is formulated as follows: "*Quando ad antecedens sequitur consequens, ad oppositum consequentis sequitur oppositum antecedentis*" (p. 185a).

only argued that any necessary proposition q is (trivially) entailed by itself and entailed by its negation, because $\neg q$ is impossible and hence, according to EIQ, $\neg q$ entails any other proposition, in particular also q[21].

Next, Pseudo-Scotus notes that (as formalized in our earlier principles DISJ 5, 6) any *tautological* disjunction like "Socrates is running, or Socrates is not running" *formally* follows from both disjuncts. Finally, he points out that in the special case where one of two contradictory propositions p or $\neg p$ "manifestly implies a contradiction," either p or $\neg p$ is *formally* entailed both by p and by p. Clearly, if, e.g., p is self-contradictory, then $\neg p$ is a tautology, and this tautology is not only formally implied, according to INF 1, by itself, but also, according to ECQ, by its negation, p[22]. Thus, 'Quaestio III' ends with the summary:

> So it is evident that the rule 'One and the same proposition does not follow from something's being so, and from its not being so' must only be understood [to hold] *for simple categorical propositions* and only in the sense of formal consequences; for as a material [i.e. simple] consequence […] the same follows from both contradictories. Similarly, if *one part of the contradictories implies a contradiction*, and also if the consequent is a *disjunction composed of contradictories*, the rule is not true[23].

Altogether, then, Aristotle's connexive theses do admit of *exceptions*. This was also recognized by Albert of Saxony (ca. 1316–1390) who, in his *Sophismata*, discussed the strange sophism "If something exists, nothing exists"[24]. It may be argued, first, that this inference cannot be valid since *de facto* something exists, so that the antecedent is true but the consequent false[25]. Furthermore,

> […] from the proposition 'Nothing exists' it follows 'Nothing exists' because a consequence from a proposition to itself is optimal. Therefore, 'Nothing exists' does not follow from 'Something exists', because otherwise the same would follow from opposites, the falsity of which principle has been maintained in the second book of the Prior Analytics[26].

[21] Cf. Duns Scotus (1891: 185a): "Probatur, prima de consequentia *simplici*, quod ad aliquam propositionem necessariam sequitur aliqua alia, ut sua aequipollens, vel aliqua hujusmodi; et illa eadem sequitur ad suam contradictoriam, quia sua contradictoria est impossibilis, modo ad impossibile sequitur quodlibet."

[22] Cf. Duns Scotus (1891: 185b): "[…] ad ambo contradictoria sequitur idem formaliter, si altera pars contradictoriarum implicet manifeste contradictionem. Probatur, quia ad illam contradictoriarum, quae non implicet contradictionem, aliquid sequitur formaliter, scilicet ipsamet […], et illa eadem sequitur ad aliam partem; quia alia pars implicet manifeste contradictionem, et ad talem sequitur quodlibet gratia formae."

[23] Cf. Duns Scotus (1891: 185b–186a): "Sic patet, quod illa regula, scilicet, *Ad idem esse, et non esse, non sequitur idem*, intelligitur solum in simplicibus Categoricis, et de consequentia formali, quia consequentia materiali, vel *ut nunc*, idem sequitur ad utrumque contradictoriarum. Similiter si altera pars contradictoriarum implicet contradictionem, vel etiam consequens sit disjunctiva composita ex contradictoriis, regula non est vera."

[24] This sophism, i.e., # XXI in the second part of Albert's collection (which comprises altogether 257 sophisms and 19 'insolubles') is very strange because the pro argument remains absolutely obscure.

[25] Cf. Albert (1975), fol. f v[ra]: "Ad sophisma respondetur quod ipsum est falsum. Nam si esset verum tunc ibi esset bona consequentia aliquid est, ergo nichil est. Modo hoc est falsum, eo quod sic est sicut significatur per antecedens et non sic sicut significatur per consequens."

[26] Cf. Albert (1975), fol. f v[ra]: "Propterea, ad istam nichil est sequitur nichil est, quia optima est consequentia eiusdem ad seipsum; ergo ad istam aliquid est non sequitur ista nichil est: quia aliter idem sequeretur ad opposita. Cuius falsitas consequentis allegata est secundo priorum."

In the further course of discussion, however, Albert considers the following objections to Aristotle's Theses:

> […] it can be proved that the same follows from opposites, for [firstly] the proposition 'Socrates is sitting' entails the disjunction 'Socrates is sitting or he is not sitting'. And this disjunction similarly follows from 'Socrates is not sitting' because each categorical proposition entails a disjunction of which it is a part. Secondly, because the necessary follows from any proposition, therefore, it follows from both opposites[27].

In view of the conclusiveness of these considerations, Albert summarizes:

> When Aristotle says in the second book of the *Prior Analytics* that the same doesn't follow from opposites, he understands this for simple categorical propositions the negations of which do not entail a contradiction. [… But] 'Socrates is sitting or he is not sitting' is not a categorical proposition, and the negation of a necessary proposition, or at least of a formally necessary proposition, entails a contradiction[28].

As this quotation – together with the other quotations in this section – shows, many prominent medieval logicians recognized that Aristotle's Theses (and their Abelardian counterparts) are only 'humbly' valid. These 'laws' fail to hold in the case of *necessary consequents* and in the case of *impossible antecedents*.

4. Is Each Proposition Compatible with Itself?

In Section 1 above, it was said that Chrysippus (ca. 280–207 BC) *maintained* that each proposition *p* is compatible with itself. As a matter of fact, however, there is no historical evidence which clearly testifies this claim. At best, one may reasonably *presume* that Chrysippus *held such an opinion*. As Sextus Empiricus reported in *Outlines of Scepticism*, four different conceptions of implication had been discussed among Stoic logicians, namely, besides *Philo*'s conception of *material* implication, and *Diodorus*'s conception of *strict* implication, two further ones:

> And those who introduce the notion of connexion say that a conditional is sound when the contradictory of its consequent is incompatible with the antecedent. [...] And those who judge by implication say that a true conditional is one whose consequent is contained potentially in the antecedent. (Kneale 1962: 129)

According to Kneale, the third account may be attributed to Chrysippus, while the fourth account is probably Peripatetic. For the sake of convenience, the third and fourth account

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[27] Cf. Albert (1975), fol. f v^{rb}: "Sed diceres probatur quod utique idem potest sequi ad opposita, nam ad istam Sortes sedet; sequitur ista disiunctiva Sortes sedet vel non sedet. Similiter sequitur ad istam Sortes non sedet, eo quod quelibet cathegorica infert disiunctivam, cuius est pars."

[28] Cf. Albert (1975), fol. f v$^{rb\text{-}va}$: "Ad ista respondetur quod quando Aristoteles secundo priorum dicit quod idem non sequitur ad opposita: intellexit hoc de propositionibus simplicibus cathegoricis, quarum contradictoriae non implicant contradictionem. […] ista Sortes sedet vel non sedet non est cathegorica: et etiam contradictorium necessarii saltem necessarium de forma est impossibile implicans contradictionem."

shall be referred to as 'Chrysippian' and 'Peripatetic' conception, no matter whether this ascription is historically correct or not.

Now, the above-quoted definition of implication, CHRYS 1, does not necessarily entail a 'hardcore' connexivist conception, because the requirement "a conditional is sound when the contradictory of its consequent is incompatible with the antecedent" might well be interpreted as defining *strict* implication[29]. What makes the Chrysippian account really *connexive* is rather the assumption, CHRYS 2, that (absolutely) *every* proposition is *compatible with itself*. In contrast to Diodorus, Chrysippus considered the example (α) "If atomic elements do not exist, then atomic elements do exist" *not* as *sound*. According to the Stoic's conception of nature, material atoms *necessarily* exist. Hence the antecedent of (α) is impossible, while its consequent is necessary. As it was argued in Lenzen (2021), Chrysippus probably denied the soundness of (α) because the negation of the consequent is *identical* – and hence *not incompatible* – with the antecedent. Generalizing from this example, McCall (2012: 415), maintained that the proponent of the connexive account of conditionals held the view that proposition p:

> […] is never incompatible with p. Accepting this in turn requires that 'compatibility' be essentially a *relational* concept, and that whether or not A is compatible with B cannot be determined by examining A and B separately. Thus even '$p \wedge \neg p$' is not incompatible with itself and 'If $p \wedge \neg p$, then not-($p \wedge \neg p$)' is connexively false[30].

A similar view was defended in the Middle Ages by an anonymous contemporary of Paul of Venice (ca. 1370–1429). Paul endorsed both principles EIQ and NAQ, so he had no qualms to acknowledge the existence of propositions which entail their own negations. But he was faced with the following objection:

> It does not follow, 'Some man is a donkey: therefore, no man is a donkey'. And this is argued according to both rules; therefore, both rules are false. […] First, because from one of two opposites the remaining one does not follow. Second, because the contradictory of the consequent stands with [i.e., is compatible with] the antecedent insofar as it is interchanged [i.e., equivalent] with that very proposition[31].

While the former objection only consists of a re-affirmation of Aristotle's Thesis, the latter is based on the Chrysippian view that if the negation of a consequent q is (identical or) *equivalent* to the antecedent p, then $\neg q$ is *compatible* with p, and hence q cannot be entailed by p. However, Paul replied:

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[29] Nowadays, it is common to consider p and q as *incompatible* if and only if the *conjunction* ($p \wedge q$) is *impossible*. Therefore, a strict implication ($p \rightarrow q$) may just be defined as $\neg\Diamond(p \wedge \neg q)$.

[30] Let it be remarked incidentally that this view was also defended, in modern times, in Nelson (1930: 443): "No property […] of a *single* proposition p is sufficient to determine whether it is consistent or inconsistent with just any other randomly selected proposition q. That is to say, from the mere fact that p is […] impossible it cannot be determined that it is inconsistent with q. The meanings of *both* propositions are required to determine the relation" (my emphasis).

[31] Paul of Venice (1984: 285); the original Latin version of the decisive passage runs as follows: "quia contradictorium consequentis stat cum antecedente eo quod convertitur cum ipso". 'Stare cum' is a standard expression used by medieval logicians to characterize the compatibility of two propositions; the opposite relation of incompatibility is often expressed by the verb 'repugnare'.

> […] I deny the inference 'the opposite of the consequent is interchanged with the antecedent; therefore, it stands with that same proposition'. Whence I say that any proposition in the world follows from [… an impossible proposition *p*] and any proposition is repugnant to that proposition. Indeed it [*p*] is repugnant to its very self because it implies the opposite of its very self. (Paul of Venice 1984: 285–6)

5. Conclusion

The main results of this paper may be summarized as follows:

- Aristotle believed that it would be *impossible* that a proposition implies, or is implied by, its own negation.
- Many medieval logicians recognized that the thesis is not strictly true: If the *antecedent* is *impossible*, then it implies its own negation, and if the *consequent* is *necessary*, then it is implied by its own negation.

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